MATH 221 : ANALYSIS I – REAL ANALYSIS

AUTUMN 2018

HOMEWORK 12

Instructor: GAUTAM BHARALI **Assigned: NOVEMBER 17, 2018**

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1. Show that the converse of the Weierstrass M-test is false.

Tip. A convincing counterexample can be constructed where the ambient metric space (X, d) is simply $\mathbb{R}$ and the target space is also $\mathbb{R}$.

2–6. Problems 3–5, 9 and 24 from "Baby" Rudin, Chapter 7. (**Caution:** The function space that we have denoted as $\mathcal{C}_b(X; \mathbb{R})$ in class is denoted by $\mathscr{C}(X)$ in Chapter 7.)

7. Work out Problem 17 from "Baby" Rudin, Chapter 7, skipping those theorems that belong to sections that we have not covered in class. Where the class $\mathcal{R}(\alpha)$ is referred to, take $\alpha = \mathsf{id}|_{[a,b]}$, where $[a, b]$ is the domain of the integrand(s) in question.

8. Let (X, d_X) be a metric space and $S \subset X$. Formulate definitions for the terms "**pointwise** equicontinuous" and "pointwise bounded" for a family $\mathscr{F}$ of functions on S taking values in a metric space (Y, d_Y).

With X and S as above, let $\mathscr{F}$ be a family of functions from S into Y that is pointwise equicontinuous and pointwise bounded. If S is compact, then show that $\mathscr{F}$ is uniformly bounded.

9. Show that there exists a sequence of polynomials $\{p_n\}$ satisfying the properties

$$p_n(x) = p_n(-x) \;\; \forall x \in \mathbb{R} \quad \text{and} \quad p_n(0) = 0,$$

for each $n = 1, 2, 3, \ldots$, such that $p_n(x) \longrightarrow |x|$ uniformly on $[-1, 1]$.