MATH 221 : ANALYSIS I – REAL ANALYSIS

AUTUMN 2018

HOMEWORK 5

Instructor: GAUTAM BHARALI **Assigned: SEPTEMBER 8, 2018**

1–3 Problems 19, 23 and 26 from Rudin, Chapter 2.
Hint. It might help to use the conclusion of Problem 4 from Homework 4.

4. Let $S \subset \mathbb{R}$ be a non-empty subset having the following property: *Given any $x, y \in S$, $x < y$, and any real number z, if $x < z < y$, then $z \in S$.* Show that S is either a bounded or an unbounded segment in $\mathbb{R}$.

5. Study the section entitled *Perfect Sets* in Chapter 2 of Rudin.

6. Fix a real number $\alpha \in (0,1)$. Let $\mathscr{C}_\alpha$ be a subset of $[0,1]$ having a construction analogous to that of the Cantor middle-thirds set. Namely, let:

$K_0 := [0,1]$,

$K_n :=$ the union of the closed intervals obtained by removing from $K_{n-1}^{(j)}$, $j = 1, \dots, 2^{n-1}$, the open interval of length $\alpha \cdot \mathsf{length}\big(K_{n-1}^{(j)}\big)$ centered at the midpoint of $K_{n-1}^{(j)}$, $n = 1, 2, 3, \dots,$

where $K_{n-1}^{(1)}, \dots, K_{n-1}^{(2^{n-1})}$ are the closed intervals whose union gives K_{n-1}. Emulate what you have studied about the Cantor middle-thirds set to show that:

(*a*) $\mathscr{C}_\alpha$ is non-empty and is a perfect set.

(*b*) the sum of the lengths of the disjoint open intervals removed from $[0,1]$ in the construction of $\mathscr{C}_\alpha$ equals 1.

7. Consider the sequence $\{a_n\}$, where

$$a_n = (n^2+1)^{1/4} - \sqrt{n+1}, \quad n = 1, 2, 3, \dots$$

Does this sequence converge? Give **justifications** for your answer, and if the sequence converges, then deduce its limit.

8. Let $\{a_n\}$ be a complex sequence that converges to A. Then, show that the sequence of arithmetic means

$$\mu_n := \frac{a_1 + \cdots + a_n}{n}, \quad n = 1, 2, 3, \dots,$$

is also convergent.
Tip. It would help to guess what $\{\mu_n\}$ converges to!