MATH 221 : ANALYSIS I – REAL ANALYSIS
AUTUMN 2018
HOMEWORK 8

Instructor: GAUTAM BHARALI **Assigned: OCTOBER 6, 2018**

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1. Read the section entitled *Discontinuities* in Chapter 4 of "Baby" Rudin.

2. Consider the function $f : [a, b] \longrightarrow \mathbb{R}$ and let $x_0 \in (a, b)$. Give a rigorous argument for the fact that f is continuous at x_0 if and only if $f(x_0+)$ and $f(x_0-)$ exist and $f(x_0+) = f(x_0-) = f(x_0)$.

3. Let S_1 and S_2 be non-empty sets and let $f : S_1 \longrightarrow S_2$. Let $\mathscr{A}$ be a non-empty subset of $\mathcal{P}(S_1)$ and let $\mathscr{B}$ be a non-empty subset of $\mathcal{P}(S_2)$. Prove the following:

$$\begin{aligned}
f\big(\cup_{A\in\mathscr{A}} A\big) &= \cup_{A\in\mathscr{A}} f(A),\\
f\big(\cap_{A\in\mathscr{A}} A\big) &\subset \cap_{A\in\mathscr{A}} f(A),\\
f^{-1}\big(\cup_{B\in\mathscr{B}} B\big) &= \cup_{B\in\mathscr{B}} f^{-1}(B),\\
f^{-1}\big(\cap_{B\in\mathscr{B}} B\big) &= \cap_{B\in\mathscr{B}} f^{-1}(B).
\end{aligned}$$

4. Let $X = [0, +\infty)$ (endowed with the usual metric). Let $n \in \mathbb{Z}_+$, and write $f_n(x) = x^n$ for each $x \in X$. Give all values of $n \in \mathbb{Z}_+$ for which $f_n : X \longrightarrow \mathbb{R}$ is **not** uniformly continuous.

5–9. Problems 6, 7 and 20–22 from "Baby" Rudin, Chapter 4.

10. Let $f : [0, 1] \longrightarrow \mathbb{R}$ be a continuous function and assume that $f(0) = f(1)$. Show that there exists a point $x_0 \in [0, 1/2]$ such that $f(x_0) = f(x_0 + \frac{1}{2})$.