MATH 224 : COMPLEX ANALYSIS
SPRING 2016
HOMEWORK 10

Instructor: GAUTAM BHARALI   **Assigned: APRIL 1, 2016**

---

1. Let Ω be a domain in $\mathbb{C}$ and let $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of injective functions that converge in $\mathcal{O}(\Omega)$ to f. Show that f is either injective or a constant function. How does the conclusion change if, instead of a domain, we allow Ω to be an arbitrary open set?

2. Let $\prod^+$ denote the upper half-plane, i.e. $\prod^+ := \{z \in \mathbb{C} : \text{Im}(z) > 0\}$. Suppose $f \in \mathcal{O}(\prod^+)$ and $f(\prod^+) \subset \mathbb{D}$. How large can $|f'(i)|$ be?

3–5. Problems 8, 9 and 14 from the exercises to III–Secn. 3 of Conway.

6. Let $\mathcal{S}$ be the strip $\mathcal{S} := \{z \in \mathbb{C} : -\pi < \text{Im}(z) < \pi\}$. Construct a biholomorphic mapping of $\mathcal{S}$ onto $\mathbb{D}$.