MATH 224 : COMPLEX ANALYSIS
SPRING 2016
HOMEWORK 3

Instructor: GAUTAM BHARALI   **Assigned: JANUARY 30, 2016**

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1. Let G be an open set in $\mathbb{C}$ and let $f : G \longrightarrow \mathbb{C}$ be a complex-valued continuous function. Let $z \in G$ and define

$$F_\varepsilon(\theta) := f(z + \varepsilon e^{i\theta}), \quad \theta \in [0, 2\pi]$$

for all $\varepsilon > 0$ sufficiently small that F_ε is defined. Prove that $F_\varepsilon \longrightarrow f(z)$ **uniformly** as $z \to 0^+$.

2–3. Problems 13 and 20 from the exercises to IV–Secn. 1 of Conway.

4–7. Problems 6, 7, 9, 11 from the exercises to IV–Secn. 2 of Conway.

8. Suppose that f is an entire function and that there exist two real numbers $M > 0$ and $p \geq 1$ such that $|f(z)| \leq M(1 + |z|^p) \; \forall z \in \mathbb{C}$. Describe, giving a **rigorous** argument, all the entire functions that satisfy this growth estimate.