MATH 224 : COMPLEX ANALYSIS

SPRING 2016

HOMEWORK 4

Instructor: GAUTAM BHARALI **Assigned: FEBRUARY 8, 2016**

---

1. Let $\Omega \subset \mathbb{C}$ be a connected open set, let $f \in \mathcal{O}(\Omega)$. Suppose f maps Ω into the circle $\{z \in \mathbb{C} : |z| = R\}$ for some $R > 0$. Prove that f is a constant.

2. Let $\Omega \subset \mathbb{C}$ be a connected open set. Then $(\mathcal{O}(\Omega), +, \cdot)$ — where $+$ and $\cdot$ denote the usual (pointwise) sum and product of functions — is a ring. Prove that $(\mathcal{O}(\Omega), +, \cdot)$ is an integral domain.

3. Let Ω_1 and Ω_2 be open subsets of $\mathbb{C}$. Let $f \in \mathcal{O}(\Omega_1)$, let $H : \Omega_2 \longrightarrow \mathbb{R}$ be harmonic on Ω_2 and suppose $f(\Omega_1) \subset \Omega_2$. Is $H \circ f$ harmonic?

4–5. Problem 9 from the exercises to IV–Secn. 3, and Problem 3 from the exercises to IV–Secn. 4 of Conway.

6. For a function $f : \mathbb{C} \longrightarrow \mathbb{C}$, we say that ***f* has two periods** if there exist two complex numbers w_1 and w_2 such that

- the pair $\{w_1, w_2\}$ is $\mathbb{R}$-linearly-independent; and
- $f(z + w_1) = f(z) = f(z + w_2)$ $\forall z \in \mathbb{C}$.

Describe the set of all entire functions having two periods.