MATH 224 : COMPLEX ANALYSIS

SPRING 2016

HOMEWORK 5

Instructor: GAUTAM BHARALI **Assigned: FEBRUARY 12, 2016**

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1. Let Ω be a domain in $\mathbb{C}$ and let $\gamma_1, \gamma_2 : [0,1] \longrightarrow \Omega$ be two piecewise smooth closed paths such that $\gamma_1(1) = \gamma_2(0)$. Show that for any point $a \notin \{\gamma_1\} \cup \{\gamma_2\}$

$$\eta(a; \gamma_1 \star \gamma_2) \;=\; \eta(a; \gamma_1) + \eta(a; \gamma_2).$$

2. Give an example of a domain $\Omega \subset \mathbb{C}$, a function $f \in \mathcal{O}(\Omega)$ and a piecewise-smooth closed curve $\gamma : [a,b] \longrightarrow \Omega$ that is **not** homologous to zero illustrating that the conclusion of Cauchy's Integral Theorem does not hold true in general for closed curves that are not homologous to zero.

In the following problems from Conway's book, replace the word "rectifiable" by the words "piecewise smooth" wherever encountered.

3–6. Problem 4 from the exercises to IV–Secn. 4, and Problems 5–7 from the exercises to IV–Secn. 5 of Conway.