MATH 224 : COMPLEX ANALYSIS
SPRING 2016
HOMEWORK 6

Instructor: GAUTAM BHARALI **Assigned: FEBRUARY 26, 2016**

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1. In class, we have established the key step in proving the following

Lemma. *Let* $\gamma_1, \gamma_2 : [0,1] \longrightarrow \Omega$ *be piecewise-smooth paths in a domain* $\Omega \subseteq \mathbb{C}$ *such that* $\gamma_1 \sim_{\text{FEP}} \gamma_2$. *Let* $\gamma_1(0) = \gamma_2(0) = a$. *Then*

$$\gamma_1 * (-\gamma_2) \sim \mathsf{const}_a,$$

i.e., $\gamma_1 * (-\gamma_2)$ *is homotopic in* Ω *to the stated constant path through closed paths.*

In class, we have constructed a homotopy $\widetilde{H}$ between const_a to some path Γ through closed paths.

a) Let $\gamma_1(1) = \gamma_2(1) = b$. Give a simple, explicit expression for Γ.

b) Using (*a*), complete the proof of the above lemma.

Hint. It is useful to remember that the relation "$\sim$" under discussion is an equivalence relation.

2. This concerns Corollary 6.17 in IV–Secn. 6.

a) Read the proof of Corollary 6.17 in IV–Secn. 6.

b) Let Ω be a simply-connected domain in $\mathbb{C}$ such that $0 \notin \Omega$. Let $a \in \Omega$. Combining the proofs of Corollaries 6.16 and 6.17, state and prove a formula, in the form of an appropriate integral, for a holomorphic branch of the logarithm on Ω that has the value 0 at a.

3. Let Ω be a domain in $\mathbb{C}$ and $f \in \mathcal{O}(\Omega)$. Show that if f is injective, then f' is non-vanishing.

4–6. Problems 5–7 from the exercises to IV–Secn. 6 of Conway.