MATH 224 : COMPLEX ANALYSIS
SPRING 2016
HOMEWORK 7

Instructor: GAUTAM BHARALI **Assigned: MARCH 3, 2016**

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1. (Carried over from Homework 6) Let Ω be a domain in $\mathbb{C}$ and $f \in \mathcal{O}(\Omega)$. Show that if f is injective, then f' is non-vanishing.

2. Let Ω be a domain in $\mathbb{C}$ and $\gamma : [0,1] \longrightarrow \Omega$ a piecewise-smooth closed path. Assume that $\gamma \approx 0$. Let

$$r := \mathsf{dist}[\{\gamma\}, \mathbb{C} \setminus \Omega] > 0.$$

Show that:

a) The set $G := \{z \in \Omega : \mathsf{dist}[z, \mathbb{C} \setminus \Omega] > r/2\}$ is an open, connected set.

b) $\eta(z;\gamma) = 0$ for each $z \in \mathbb{C} \setminus G$.

3. Let $f(z) = 1/(z-1)(z-2)$.

a) Using **purely elementary** methods (i.e., do **not** appeal to the integral formula for the Laurent coefficients), compute the Laurent expansions of f around $z = 1$ and $z = 2$.

b) Give the regions in which these two expansions are absolutely convergent.

4. Suppose f is holomorphic in the punctured disc $D(a;\delta)^*$ and suppose a is a pole of f of order $m \geq 1$. Then, show that

$$\mathrm{Res}(f;a) = \frac{1}{(m-1)!} g^{(m-1)}(a),$$

where $g(z) := (z-a)^m f(z) \ \forall z \in D(a;\delta)^*$.

5. Problem 13 from the exercises to V–Secn. 1 of Conway.

Note. This problem is **important.** Firstly, it introduces certain new definitions. Secondly, it contains the fundamental idea showing that the one-point compactification of $\mathbb{C}$ (what is it homeomorphic to ?) is a complex manifold.

6. Carefully review Examples 2.5, 2.7, 2.10 & 2.12 in Chapter V of Conway to understand four important types of tricks for applying the residue theorem to evaluate **real integrals.**

7–11. Problems 2, 6, 14, 15 and 16 from the exercises to V–Secn. 1 of Conway.

12. Describe all the injective holomorphic maps from $\mathbb{C} \setminus \{0\}$ **onto** itself.