MATH 224 : COMPLEX ANALYSIS

SPRING 2016

HOMEWORK 9

Instructor: GAUTAM BHARALI **Assigned: MARCH 25, 2016**

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1. Let $f \in \mathcal{O}(\mathbb{D})$ and assume that $|f(z)| \leq 1$ for every $z \in \mathbb{D}$, where $\mathbb{D}$ denotes the open unit disc with centre at $0 \in \mathbb{C}$. Show that

$$\frac{|f(0)| - |z|}{1 - |f(0)||z|} \leq |f(z)| \leq \frac{|f(0)| + |z|}{1 + |f(0)||z|} \quad \forall z \in \mathbb{D}.$$

2. Let Ω be a domain in $\mathbb{C}$, and let $\{f_n\} \subset \mathcal{O}(\Omega)$. Suppose this sequence has the property that for each circle $\mathsf{C} \subset \Omega$, there exists a function $f_{\mathsf{C}} : \mathsf{C} \longrightarrow \mathbb{C}$ such that $f_n|_{\mathsf{C}} \longrightarrow f_{\mathsf{C}}$ **uniformly** on C. Prove that there exists a holomorphic function F such that $f_n \longrightarrow F$ uniformly on compact subsets of Ω.

3–4. Problems 6 and 7 from the exercises to VII–Secn. 2 of Conway.