MATH 380 : INTRODUCTION TO COMPLEX DYNAMICS

AUTUMN 2016

HOMEWORK 2

Instructor: GAUTAM BHARALI **DUE: Friday, Sep. 23, 2016**

Remarks and instructions :

- You are allowed to discuss these problems with your fellow-students, but individually-written and **original** write-ups are expected for submission.

b) We shall use the following notation:

$$\mathbb{D} := \text{the open unit disc with centre } 0 \in \mathbb{C},$$
$$\widehat{\mathbb{C}} := \text{the one-point compactification of } \mathbb{C}.$$

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1. Show that

$$h_{\mathbb{D}}(z_1, z_2) := \left| \frac{z_1 - z_2}{1 - \overline{z}_2 z_1} \right| \quad \forall z_1, z_2 \in \mathbb{D}$$

is a metric on $\mathbb{D}$.

2. Let X be a hyperbolic Riemann surface, and let Δ_X denote the Poincaré distance on X. Show that the topology induced by Δ_X is the same as the manifold topology on X.

3. Show that there is no non-constant holomorphic map from $\widehat{\mathbb{C}}$ or from $\mathbb{T}^2$ (regardless of the complex structure on it) into any hyperbolic Riemann surface.

4. Show that any holomorphic map $f : \widehat{\mathbb{C}} \longrightarrow \widehat{\mathbb{C}}$ must be a rational map.

5. Let $f : \widehat{\mathbb{C}} \longrightarrow \widehat{\mathbb{C}}$ be a holomorphic map and let z_0 be a fixed point of f. Let $\lambda_f(z_0)$ denote the multiplier of f. Let ψ be any Möbius transformation such that $\psi(z_0) \neq \infty$. Show that:

a) $\psi \circ f \circ \psi^{-1}$ is holomorphic in the classical sense in a small planar neighbourhood of $\psi(z_0)$.

b) $\lambda_f(z_0) = (\psi \circ f \circ \psi^{-1})'(\psi(z_0))$.

6. Let $f : \widehat{\mathbb{C}} \longrightarrow \widehat{\mathbb{C}}$ be a non-constant holomorphic map. Suppose f has a repelling periodic orbit $(z_0, z_1, \dots, z_{p-1})$. Show that each z_j lies in the Julia set of f, $j = 0, 1, \dots, p-1$.