# MATH 380 : INTRODUCTION TO COMPLEX DYNAMICS

## AUTUMN 2016

## HOMEWORK 4

**Instructor: GAUTAM BHARALI** **DUE: Friday, Nov. 25, 2016**

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**Remarks and instructions :**

- You are allowed to discuss these problems with your fellow-students, but individually-written and **original** write-ups are expected for submission.

b) We shall use the following notation:

$$\widehat{\mathbb{C}} := \text{the one-point compactification of } \mathbb{C}.$$
$$\dim_H(S) := \text{the Hausdorff dimension of } S \subseteq \mathbb{R}^n \text{ (fixing some } n \in \mathbb{Z}_+\text{)}.$$

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**1.** Let $f : \widehat{\mathbb{C}} \longrightarrow \widehat{\mathbb{C}}$ be a non-constant holomorphic map and let $z_0$ be repelling fixed point. Formulate and prove, in this case, an analogue of the **global** Koenig semi-conjugacy theorem that was proved in class.
**Hint.** Recall that there is a neighbourhood $U \ni z_0$ such that $f|_U$ is invertible.

**2.** Fix $n \in \mathbb{Z}_+$. Show that for any set $S \subseteq \mathbb{R}^n$, the $\alpha^{th}$ Hausdorff outer measure $H_\alpha(S) = 0$ for every $\alpha > n$.
**Note.** You can use *without proof* any statement about $H_\alpha$ presented prior to the above in class.

**3.** Fix $n \in \mathbb{Z}_+$. Show that for any non-empty open set $U \subseteq \mathbb{R}^n$, $\dim_H(U) = n$.

**4.** Let $f : \widehat{\mathbb{C}} \longrightarrow \widehat{\mathbb{C}}$ be a non-constant holomorphic map and let $\deg(f) \geq 2$. Given any $\varphi \in \mathcal{C}(\widehat{\mathbb{C}}; \mathbb{C})$, define

$$T_f[\varphi](w) := \sum_{w \in f^{-1}\{w\}^\bullet} \varphi(z),$$

where the notation $f^{-1}\{w\}^\bullet$ denotes the **list** of roots of the equation $f(z) = w$ repeated according to multiplicity. Show that $Tf[\varphi]$ is continuous on $\widehat{\mathbb{C}}$.
**Hint.** Do you see a covering space somewhere in this set-up ?