UM 204 : INTRODUCTION TO BASIC ANALYSIS

SPRING 2019

HOMEWORK 12

Instructor: GAUTAM BHARALI **Assigned: APRIL 6, 2019**

---

1. Refer to the definition of the *exponential function* — denoted either as $\exp(x)$ or e^x for any $x \in \mathbb{R}$ — introduced in class. Show that for any number $r \in \mathbb{R}$,

$$x^r = e^{r\log(x)} \quad \forall x > 0$$

(refer to Homework 5 for the definition of x^r).
Hint. **Fix** a point $x_0 > 0$. Consider the two auxiliary functions $\varphi_1, \varphi_2 : \mathbb{R} \longrightarrow \mathbb{R}$:

$$\varphi_1(r) := x_0^r \quad \text{and} \quad \varphi_2(r) := \exp\big(r\log(x_0)\big).$$

Further, consider $\varphi_1|_{[n,\,n+1]}$, $\varphi_2|_{[n,\,n+1]}$, $n = 0, \pm 1, \pm 2, \dots$

2. Does

$$\lim_{x\to 0} \frac{1}{x} \int_{|x|^{3/2}}^{x} \exp(t^3)\, dt$$

exist? **Justify** your answer.

3. Let (Z, d) be a metric space and let $\xi \in Z$. Show that the function $Z \ni z \longmapsto d(z, \xi)$ is continuous.

4–6. Problems 3–5 from "Baby" Rudin, Chapter 7.

7. Let X and Y be metric spaces and let $\{f_n\}$ be a sequence of Y-valued functions. Assume that there is a function $f : X \longrightarrow Y$ such that $f_n \to f$ uniformly. Let a be a limit point of X. Show that for any sequence $\{x_n\} \subset X \setminus \{a\}$ that converges to a, $f_n(x_n) \to f(a)$ as $n \to \infty$.