UM 204 : INTRODUCTION TO BASIC ANALYSIS

SPRING 2019

HOMEWORK 13

Instructor: GAUTAM BHARALI **Assigned: APRIL 15, 2019**

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1. Prove the following generalization of the **Weierstrass $\boldsymbol{M}$-test** (see Chapter 7 of "Baby" Rudin for a statement of the classical M-test):
Let X be a metric space and $S \subseteq X$ a non-empty subset. Let $(V, \|\cdot\|)$ be a normed vector space over $\mathbb{R}$ or $\mathbb{C}$ that is complete with respect to the metric induced by $\|\cdot\|$. Let $\{f_n\}$ be a sequence of V-valued functions. Suppose that, for each n, there exists a constant $M_n > 0$ such that

$$\|f_n(x)\| \leq M_n \quad \forall x \in S, \text{ and } n = 1, 2, 3, \dots,$$

such that the series $\sum_{n=1}^{\infty} M_n$ is convergent. Then, the series $\sum_{n=1}^{\infty} f_n$ is uniformly convergent.

2–4. Problems 11, 14 and 15 from "Baby" Rudin, Chapter 7.

5. Show that there exists a sequence of polynomials $\{p_n\}$ satisfying the properties

$$p_n(x) = p_n(-x) \;\; \forall x \in \mathbb{R} \quad \text{and} \quad p_n(0) = 0,$$

for each $n = 1, 2, 3, \dots$, such that $p_n(x) \longrightarrow |x|$ uniformly on $[-1, 1]$.