UM 204 : INTRODUCTION TO BASIC ANALYSIS
SPRING 2019
HOMEWORK 1

Instructor: GAUTAM BHARALI **Assigned: JANUARY 11, 2019**

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1. Show that for any natural number n, $S(n) \neq n$. (Here, $S(\cdot)$ denotes the successor as postulated by Peano's axioms.)

2. Prove, using Peano's axioms, that if $P(n)$ denotes some property involving the natural number n, and if

- $P(1)$ is true, and
- whenever $P(n)$ is true, then $P(S(n))$ is true,

then $P(n)$ is true for every natural number $n \neq 0$.

3. Recall the binary operations $+$ and $\cdot$ given by Peano arithmetic. Prove that both operations are commutative.

4. Let $\mathscr{F}$ be a non-empty set of sets. Recall that we defined $\bigcap_{S \in \mathscr{F}} S$ by choosing a set $A \in \mathscr{F}$ and appealing to the Axiom of Specification to define $\bigcap_{S \in \mathscr{F}} S$ as a certain subset of A. Show that the set $\bigcap_{S \in \mathscr{F}} S$ does not depend on the choice of A.

5. Let X be a set and let $\mathscr{F}$ be a non-empty set of subsets of S. Prove the following (which is one of the two **de Morgan laws**)

$$X \setminus \Big(\bigcap_{S \in \mathscr{F}} S \Big) = \bigcup_{S \in \mathscr{F}} (X \setminus S).$$

6. Let X and Y be two non-empty sets. For $a \in X$ and $b \in Y$, recall that the ordered pair (a, b) is the abbreviation for a certain set. Show, mentioning **clearly** the axioms to which you are appealing, that if $(a, b), (x, y) \in X \times Y$, then $(a, b) = (x, y)$ if and only if $a = x$ and $b = y$.