UMA 101 : ANALYSIS & LINEAR ALGEBRA – I

AUTUMN 2023

HOMEWORK 5

Instructor: GAUTAM BHARALI **Assigned: SEPTEMBER 5, 2023**

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1. Let $\{a_n\}$ be a convergent sequence with limit L. Prove that the sequence $\{b_n\}$, where

$$b_n \,=\, \frac{a_1 + \cdots + a_n}{n},$$

converges to L.

2. Let $\sum_{n=1}^{\infty} a_n$ be a convergent real series. Prove that $\lim_{n\to\infty} a_n = 0$.
Hint. Apply the lemma discussed during the September 4 lecture to an appropriate sequence.

3. Determine whether or not each of the following non-negative series converges. Give **justifications.**

(*a*) (Apostol, 10.14, Prob. 1) $\sum_{n=1}^{\infty} n/(4n-3)(4n-1)$

(*b*) $\sum_{n=1}^{\infty} |\sin(5n^2)|/n^2$

(*c*) $\sum_{n=1}^{\infty} \big(3+(-1)^n\big)/3^n$

(*d*) (Apostol, 10.14, Prob. 7) $\sum_{n=1}^{\infty} n!/(n+2)!$

(*e*) $\sum_{n=1}^{\infty} b_n/5^n$, where $\{b_n\}$ is a bounded sequence with non-negative terms

(*f*) $\sum_{n=1}^{\infty} (n^2+(-1)^n)/n^2$

4. State whether or not each of the following non-negative series converges. Give **justifications.**

a) (Apostol, 10.16, Prob. 13) $\sum_{n=1}^{\infty} \frac{n^3\left(\sqrt{2}+(-1)^n\right)^n}{3^n}$

b) $\sum_{n=1}^{\infty} (n!)^2/(2n)!$

Note. You must use **only** the tests and results discussed in class or assigned for self-study.