UMA 101 : ANALYSIS & LINEAR ALGEBRA – I

AUTUMN 2023

HOMEWORK 7

Instructor: GAUTAM BHARALI **Assigned: SEPTEMBER 19, 2023**

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1. Solve:

- Problems 1, 3, 4, 6, 7, and 10 from Section 10.14 of Apostol's book, and
- Problems 1, 3, 5, and 8 from Section 10.16 of Apostol's book.

Make free use, **without** any proof, of the fundamental limits (10.9), (10.10), and (10.12) — given on page 380 of Apostol's book — and any limits presented in class, if the need arises.

2. Prove the following:

Theorem. Let $\sum_{n=1}^{\infty} a_n$ be a series with non-negative terms such that $a_{n+1} \leq a_n$ for $n = 1, 2, 3, \dots$. Then, the given series converges if and only if the series

$$\sum_{k=0}^{\infty} 2^k a_{2^k}$$

converges.

3. Prove the following:

Theorem (*Squeeze Theorem*)**.** Suppose $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ are real sequences and that there exists a number $M \in \mathbb{P}$ such that

$$a_n \ \leq \ b_n \ \leq \ c_n \quad \forall n \geq M.$$

Suppose $\{a_n\}$ and $\{c_n\}$ converge to L. Then $\{b_n\}$ also converges to L.

4. Let $f(x) = [x]$, i.e., the greatest integer function. Fix a point $p \in \mathbb{R}$. Using either of the definitions of the limit of $f(x)$ as x approaches p — whichever is more convenient — discuss whether or not $f(x)$ has a limit as x approaches p, depending on p.