Problem set 6

Problem 1. Find the density of $X+Y$.

1. $X \sim \text{Gamma}(\alpha_1, \lambda)$ independent of $Y \sim \text{Gamma}(\alpha_2, \lambda)$.

2. X, Y are i.i.d Uniform$[0,1]$.

3. $X \sim N(\mu_1, \sigma_1^2)$ indpendent of $Y \sim N(\mu_2, \sigma_2^2)$.

Problem 2. If X, Y are i.i.d. $N(0,1)$, find the distribution of X^2+Y^2. What about the sum of squares of n i.i.d. $N(0,1)$ random variables?

Problem 3. If X, Y are i.i.d. Exp(1), find the joint density of $U = X+Y$ and $V = Y$.

Problem 4. Let X, Y be independent integer-valued random variables. If X has pmf f and Y has pmf g, then $X+Y$ has pmf h given by $h(k) = \sum_{i \in \mathbb{Z}} f(i)g(k-i)$. Use this to show the following.

1. If $X \sim \text{Bin}(n, p)$ and $Y \sim \text{Bin}(m, p)$, then $X+Y \sim \text{Bin}(m+n, p)$.

2. If $X \sim \text{Pois}(\lambda)$ and $Y \sim \text{Pois}(\mu)$, then $X+Y \sim \text{Pois}(\lambda+\mu)$.

3. If X, Y are i.i.d with Geo(p) distribution, then $X+Y$ has a negative binomial distribution with parameters 2 and p.

Problem 5. Let X, Y be i.i.d. random variables with density f. Let $U = \min\{X, Y\}$ and $V = \max\{X, Y\}$.

1. Find the joint density of (U, V).

2. If X, Y are i.i.d. Exp(λ), show that $U \sim \text{Exp}(2\lambda)$.

3. If X, Y are i.i.d. Uniform$[0,1]$, find the density of V explicitly.

Problem 6. (*) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with CDF F and density f. Let $M = \max\{X_1, \ldots, X_n\}$.

1. Show that the CDF of M is given by $G(x) = (F(x))^n$ and its pdf is given by $g(x) = n(F(x))^{n-1} f(x)$.

2. What are the CDF and pdf of $M' = \min\{X_1, \ldots, X_n\}$?