Part 1

In page number 2 , Example 3 , the set theoretic definition of Ω is incorrect. The given definition stands for set of all paths.

In page number 4 , Example 6 , line 2 , it is written $\sum_k |I_k| = 4\epsilon$, it should be $\sum_k |I_k| = 2\epsilon$.

In page number 6 , footnote 2 , line 2 , the bijection is given as $\sigma : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ it should be $\sigma : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$.

In page number 8 , after definition 16 , when showing that the two sigma algebras are equal some equations are missing or incorrect. The incorrect one (i.e it is not wanted) is $(a, b] = \cap_n (a, b + \frac{1}{n}]$ while it should have been $(a, b] = \cap_n (a, b + \frac{1}{n})$. The missing ones are $[0, b] = \cap_n [0, b + \frac{1}{n})$ and $[0, b) = \cup_n [0, b - \frac{1}{n}]$.

In page number 12 , proof of Claim 25 , last paragraph , first line , it is written as $\lambda_*(A) \geq \lambda_*(A_n)$, it should be $\lambda_*(A) \geq \lambda_*(\cup_{k \leq n} A_k)$.

In page number 12 , proof of Claim 26 , line 1 , it is written "let $\{I_n\}$ be an open cover such that $\lambda_*(E) \geq \sum |I_n|$" but this is not true in general. The proof goes through by using an ϵ of room.

In page number 16 , in the first construction of a non-measurable set , second paragraph , third line from the end , it is written as "$x \in A + r$ where $r = y - x$ or $y - x + 1$", it should be $x \in A + r$ where $r = x - y$ or $x - y + 1$.

In page number 16 , in the second construction showing λ_* is not finite additive , in line number 3-4 , it is written that the example is stronger than the previous one. That is not true. Existence of a non-measurable set is equivalent to λ_* being not finitely additive because of the following claim.

Claim 1. *Let $A_n, A \in 2^{[0,1]}$ such that $A_n \uparrow A$, then $\lambda_*(A_n) \uparrow \lambda_*(A)$.*

Given the claim, finite additivity of λ_* will be extended to countable additivity and hence there would not exist a non measurable set.

Proof. The idea is to use liminf of suitable measurable sets and to use the countable additivity of λ_* on the set of measurable sets. By definition of outer measure, for every set A_n there exists an open cover $\{I_n\}$ such that $\sum \lambda_*(I_k) \leq \lambda_*(A_n) + \epsilon$ for some $\epsilon > 0$. Let $J_n = \cup_k I_k$ so that we have

$\lambda_*(J_n) \leq \lambda_*(A_n)+\epsilon$. Note that $A_n \subseteq J_n$ and hence we have $\lambda_*(A_n) \leq \lambda_*(J_n)$. So overall we have for every n , $\lambda_*(A_n) \leq \lambda_*(J_n) \leq \lambda_*(A_n)+\epsilon$. Now consider $L_n = \cap_{k\geq n} J_k$, $L_n \uparrow L$ and $A_n \subseteq L_n$ for all n. Now $\lambda_*(L_n) \leq \lambda_*(J_n)$ so that combining we get $\lambda_*(A_n) \leq \lambda_*(L_n) \leq \lambda_*(J_n) \leq \lambda_*(A_n)+\epsilon$. Note also that L_n are measurable and thus $\lambda_*(L_n) \uparrow \lambda_*(L)$. Putting it all together we get $\lambda_*(A_n) \leq \lambda_*(A) \leq \lambda_*(L) \leq \lambda_*(L_n)+\delta \leq \lambda_*(A_n)+\epsilon+\delta$ where $\delta \to 0$ as $n \to \infty$. Hence the result follows. $\square$

In page number 17 , line 1 , the definition of the group is given as $G = \{n\alpha : n \in \mathbb{Z}\}$, it should be $G = \{n\alpha(\text{mod}1) : n \in \mathbb{Z}\}$.

In page number 17 , proof of Steinhaus' lemma , in the equations second line , it is written $2\mu(A)$ whereas it should be $2\mu(A')$ and the next line is given as an equality, it should be an inequality.

In page number 19 , after Example 38 , in the proof of Z = X + Y being a random variable the argument is invalid for we can take for some $\omega \in \Omega, X(\omega) = \sqrt{(2)}, Y(\omega) = 1 - \sqrt{(2)}$ and $Z(\omega) = 1$ when $t = 1$, then the union given on the right hand side does not contain ω. It is better to work with open intervals, then the same argument will go through.

In page number 22 , Example 45 , the inequalities given for the variable in the definition of the CDF is wrong.

In page number 24 , proof of theorem 42 , last paragraph, line 1 , it is claimed that T^{-1} is Borel-measurable but this has not been shown. What has been shown is that $T(\mathbb{R}^2)$ is Borel. We need to show that T takes Borel sets to Borel sets. I did not get this.

In page number 24 , last but one line it is written as $\mu = \lambda \circ T \circ S^{-1}$ whereas it should be $\mu = \lambda \circ S^{-1} \circ T$ and the map is given as $T \circ S : (0,1) \to \mathbb{R}^2$ whereas it should be $T^{-1} \circ S : (0,1) \to \mathbb{R}^2$.

In page number 25 , Example 48 , (2) Gamma distribution , the density has $\lambda^{\alpha-1}$ rather than λ^{α}.

In page number 25 , Example 48 , (4) Beta distribution , the definition of $B(a, b)$ given is the reciprocal of the actual definition $\Gamma(a)\Gamma(b) = \Gamma(a + b)B(a, b)$.

In page number 25 , Example 49 , line 2-3 , it is written "p is called the pmf(probability density function)". It should be probability mass function.

In page number 27 , Proof of Proposition 55 , line 3 - 4 it is written $\liminf F_{\mu_n}(x) \geq F_\mu(x-u)$, it should be $\liminf F_{\mu_n}(x) \geq F_\mu(x-u) - u$.

In page number 28 , Proof of Proposition 55 , second part of the proof (line number 3 in page 28), the hypothesis is wrongly stated. It should be $|F(x)| < u \ \forall \ x \leq x_1$ and $|F(x)| > 1 - u \ \forall \ x \geq x_m$ and $|F_n(x_i) - F(x)| < u \ , \forall \ i, \ \forall \ n \geq N$ for some $N \in \mathbb{N}$ and in the last line of the proof it is written $d(\mu_n, \mu) \leq u$ whereas it should be $d(\mu_n, \mu) \leq 2u$.

In page number 29 , Proof of Lemma 58 , first paragraph , line 5 - 6 , it is written $F_{n_{l,j}}(x_j) \to \alpha_j$ for each $j \leq l$, it should be $F_{n_{l,k}}(x_j) \to \alpha_j$ for each $j \leq l$.

In page number 34 , Example 68 , line 1 , the definition of $f_n(t)$ is given as $-\frac{1}{n}\mathbf{1}_{\mathbf{t}\leq\frac{1}{n}}$, it should be $f_n(t) = -n\mathbf{1}_{\mathbf{t}\leq\frac{1}{n}}$.

In page number 38 , Remark 77 , line 4 - 5 , it is written "defines an inner product on L^p, it should be defines an inner product on L^2.

In page number 41 , Example 87 , last but one line , it is written as "We get $\int_0^u e^{-u} du = ue^{-u}$". The variable of integration should be changed . Also is it the Gamma(1,1) density because $\alpha = 2, \lambda = 1$.

In page number 42 , Example 88 , (3) , line 1 it is written $g(t) = \int_{\mathbb{R}} f(t-v, v)dx$. The variable of integration should be v. In line 3 , it is written "Hence by corollary 86" , it is actually Proposition 86.

In page number 43 , Definition 92 , for absolutely continuous the notation is given as $\mu \ll \mu$, it should be $\mu \ll \nu$.

In page number 44 , statement of Theorem 95 , it is written as $\mu \ll \nu$ whereas it should be $\nu \ll \mu$

In page number 44 , Example 98 , line 2, it is written as $X_\lambda(\omega) = \sum_{k=1}^{\infty} \lambda^{-k} X_k(\omega)$ whereas it should be $X_\lambda(\omega) = \sum_{k=1}^{\infty} \lambda^{-k} B_k(\omega)$.

In page number 46 , Lemma 101 , proof , line 2 , it is written $f_n \uparrow \mathbf{1}_{[\mathbf{a},\mathbf{b}]} \ldots$ and the caution says the same thing is not true , it should have been (a, b). Two lines after the expression for $g(x)$ has been given it is written $G(x) = \int_{-\infty}^{x} g(u)du$, the constant C has been forgotten.

In page number 47 , before Exercise 103 , after definition of correlation , it is written "A correlation of 1 implies X = Y a.s while a correlation of -1 implies X = -Y a.s", it should have been A correlation of 1 implies $X = cY + d$ for some $c > 0$ while a correlation of -1 implies $X = cY + d$ for some $c < 0$. This is so as $Cor(X, Y) = Cor(aX + b, cY + d)$ when $ac > 0$ follows from $Cov(aX+b, cY+) = acCov(X, Y)$ and $Var(aX+b) = a^2 Var(X)$.

In page number 47 , Example 104 , the mean and variance of the exponential of the exponential distribution are given as λ and λ^2, it should have been λ^{-1} and λ^{-2}. And in the same line the given even moments are for the standard normal distribution.

In page number 48 , in (1) , line 2 , it is written $(A \times B) \cap (A_2 \cap B_2) = (A_1 \cap A_2) \times (B_1 \cap B_2)$, it should have been $(A_1 \times B_1) \cap (A_2 \times B_2) = (A_1 \cap A_2) \times (B_1 \cap B_2)$.

In page number 50 , Example 110 , last but one line , in the second of the iterated integrals the domains have bben switched.

In page number 54 , proof of Lemma 119 , part (2) , line 1 , it is written $\mathcal{G} := \sigma(X_i)$, it should be $\mathcal{G} := \sigma(Z_i)$.

In page number 56 , proof of Proposition 122 , (2) , line 3 , the definition is written as $U_1 := X_1/2 + X_3/2^3 + X_5/2^5 + \ldots$, it should be $U_1 := X_1/2 + X_3/2^2 + X_5/2^3 + \ldots$.

Regarding an example of a measure which is not a push forward of the Lebesgue measure on [0,1], we can take $(\{0,1\}^{\{0,1\}^{\mathbb{R}}}, \otimes_i \mathcal{F}_i, \otimes_i \text{Ber(p)})$ $p \neq 0, 1$. A cardinality argument will do. Not interesting since I do not know of any useful space with the above cardinality. For the space $(\{0,1\}^{\mathbb{R}}, \otimes_i \mathcal{F}_i, \otimes_i \text{Ber(p)})$ $p \neq 0, 1$, to show that the measure is not a push forward of the Lebesgue measure on [0,1] is equivalent to showing the non existence of uncountable independent Ber(p) random variables on [0,1]. This I have not been able to show. To get an example on [0,1] be changing the σ-algebra seems unlikely as the Borel σ-algebra is big. I do not know of another σ-algebra with a measure on it that is different from the Borel σ-algebra in the sense that it contains a rich enough class of sets not present in the latter. We could take the Lebesgue σ-algebra and whether $(\mathcal{B}, \lambda)$ could be pushed to $(\overline{\mathcal{B}}, \lambda)$ is also interesting. Note that in neither this case nor the case of $\{0,1\}^{\mathbb{R}}$, the map which pushes forward $(\mathcal{B}, \lambda)$ is surjective.

Part 2

In page number 3 , Remark 2 , last line , it is written $\mathbf{P}(X > t) = \mathbf{P}(e^{\lambda X} < e^{\lambda t})$, it should be $\mathbf{P}(X > t) = \mathbf{P}(e^{\lambda X} > e^{\lambda t})$.

In page number 5 , Borel-Cantelli lemmas , last calculation when applying the second moment method , it is written $\mathbf{P}(X \geq 1) \geq \ldots$, it should be $\mathbf{P}(X > 0) \geq \ldots$.

In page number 6 , after equation number (1) , line 1 , it is written "observe that $e^x - (1 - x)$ is equal to ..." , it should be $e^{-x} - (1 - x)$. In the proof for the second inequality, the inequalities have been reversed and the summation is not proper. It is : if $|x| < \frac{1}{2}$, then $\log(1 - x) \geq -x - x^2 + \frac{1}{2}x^2 - \frac{1}{2}\sum_{k=3}^{\infty} |x|^k \quad \geq -x - x^2$ since $\sum_{k=3}^{\infty} |x|^k \leq x^2 \sum_{k=1}^{\infty} 2^{-k} = x^2$.

Part 2

In page number 3 , Remark 2 , last but one line , it is written $\mathbf{P}(X > t) = \mathbf{P}(e^{\lambda X} e^{\lambda t})$, it should be $\mathbf{P}(X > t) = \mathbf{P}(e^{\lambda X} > e^{\lambda t})$.

In page number 6 , after equation number (1) , in the proof for the second inequality, the first equation , the power of $|x|$ has been fixed at 3, it should be k.

In page number 7 , first line , it is written $\mathbf{E}[X_{t,k}] = (1 - 1/n)^k$, it should be $\mathbf{E}[X_{t,k}] = (1 - 1/n)^t$ and similarly for the next formula.

In page number 7 , equation (4) , the last inequality is written as $ne^{-\frac{n \log n + n\theta_n}{n}} \leq e^{-\theta_n}$, it should be $ne^{-\frac{n \log n + n\theta_n}{n}} \leq ne^{-\theta_n} = o(1)$.

In page number 7 , first line after equation (4) , it is written $t < n \log n - n\theta_n$, it should be $t = n \log n - n\theta_n$.

In page number 7 , after equation (4), the inequalities written for $\mathbf{E}[S_{t,n}^2]$ and $\mathbf{E}[S_{t,n}]$ are not correct exactly , they miss factors of n^2 and n respectively , though this does not change the final result.

In page number 7 , last set of equations , first equation , equal to sign is missing.

In page number 8 , before section 3.3 , in the last equation the bound for variance is missing the constant factor C.

In page number 14 , before the last equation , there is reference to equation (6.1) which has not been named.

In page number 19 , Example 24 , first paragraph , it is written $\mathbf{E}[|X_i| \mathbf{1}_{|X_i|>t}] \leq t^{-(p-1)} M$. I could not prove this. By Markov , we have $t\mathbf{P}\{|X_i| > t\} \leq t^{-(p-1)} M$ but from positivity of expectation we get $\mathbf{E}[|X_i| \mathbf{1}_{|X_i|>t}] \geq \mathbf{E}[t \mathbf{1}_{|X_i|>t}] = t\mathbf{P}\{|X_i| > t\}$, opposite inequalities.

In page number 19 , Example 24 , second paragraph , the example given for failure is example 22 , but the r.v.s in example 22 are not L^1 bounded . Perhaps what was meant was $X_n = n \mathbf{1}_{[0,1/n]}$.

In page number 22 , first line , there is reference to equation (9), it should be to the previous equation.

In page number 24 , in the proof of Lemma 34 , second line , we use convexity of exponential on $\mathbb{R}$ not just on $[-1,1]$. Also the equation written is

$$e^{\lambda X} = \frac{1}{2}\left(\left(1+\frac{X}{d}\right)e^{-\lambda d} + \left(1-\frac{X}{d}\right)e^{\lambda d}\right)$$

it should be

$$e^{\lambda X} = \frac{1}{2}\left(\left(1+\frac{X}{d}\right)e^{\lambda d} + \left(1-\frac{X}{d}\right)e^{-\lambda d}\right)$$

In page number 24 , last paragraph , (1) large deviation regime , in the equations written the middle term should be $\mathbf{P}(|S_n - \mathbf{E}[S_n]| \geq nu)$.

In page number 25 , (2) moderate deviation regime , in the left hand side of the equation the δ should be replaced by t.

In page number 25 , (2) moderate deviation regime , last line , it is written $\mathbf{P}(|S_n - \mathbf{E}[S_n]| \geq un^{\alpha}) \leq 2e^{-\frac{u^2}{2}n^{2\alpha-1}}$, it should be $\mathbf{P}(|S_n - \mathbf{E}[S_n]| \geq un^{\alpha}) \leq 2e^{-\frac{u^2}{8d^2}n^{2\alpha-1}}$ (the d is missing).

In page number 25 , two lines before section 12 , it is written "$I(u)$ is larger than $u^2/2d$ which ..." , should it not be $I(u)$ is larger than $u^2/2d^2$ since that is what Hoeffding's inequality gives us.

In page number 26 , in the proof of theorem 37 , after equation (11) , second line , it is written "For a fixed k we can use Chebyshev's to get ..." but what follows is the maximal inequality.

In page number 26 , Remark 38 , second paragraph, third line , it is written $Y_n = X_n \mathbf{1}_{|X_n|>A}$, it should be $Y_n = X_n \mathbf{1}_{|X_n|\leq A}$.

In page number 27 , proof of Lemma 40 , equation (13), the sign of the last term is given as $-$ it should be $+$ (Does not matter since it is anyway 0).

In page number 29 , before exercise 42 , to show CLT for Exponential, in the last of the equations a factor of 2 is missing (the distribution of N(0,1)).

In page number 31 , last line , the definition of $R_n(t)$ is missing $n^{-3/2}$.

In page number 32 , 16.2 , proof , note that V_{k+1} has X_k and X_{k+1} while U_k has neither of them. The sequence of equations should be

$$\mathbf{E}[f(Y_1)] - \mathbf{E}\left[f\left(\frac{1}{\sqrt{n}}S_n^X\right)\right] = \sum_{k=0}^{n-1}\mathbf{E}[f(V_k) - f(V_{k+1})]$$

$$= \sum_{k=0}^{n-1}\mathbf{E}[f(V_k) - f(U_{k+1})] - \sum_{k=0}^{n-1}\mathbf{E}[f(V_{k+1}) - f(U_{k+1})]$$

and the corresponding Taylor explansions will be

$$f(V_k) - f(U_{k+1}) = f'(U_{k+1})\frac{Y_{k+1}}{\sqrt{n}} + f''(U_{k+1})\frac{Y_{k+1}^2}{2n} + f'''(U_{k+1}^*)\frac{Y_{k+1}^3}{6n^{\frac{3}{2}}}$$

$$f(V_{k+1}) - f(U_{k+1}) = f'(U_{k+1})\frac{X_{k+1}}{\sqrt{n}} + f''(U_{k+1})\frac{X_{k+1}^2}{2n} + f'''(U_{k+1}^{**})\frac{X_{k+1}^3}{6n^{\frac{3}{2}}}$$

and U_{k+1} is independent of X_{k+1} and Y_{k+1} and .... An alternative way is the following set

$$\mathbf{E}[f(Y_1)] - \mathbf{E}\left[f\left(\frac{1}{\sqrt{n}}S_n^X\right)\right] = \sum_{k=1}^{n}\mathbf{E}[f(V_{k-1}) - f(V_k)]$$

$$= \sum_{k=1}^{n}\mathbf{E}[f(V_{k-1}) - f(U_k)] - \sum_{k=1}^{n}\mathbf{E}[f(V_k) - f(U_k)]$$

and the corresponding Taylor explansions will be

$$f(V_{k-1}) - f(U_k) = f'(U_k)\frac{Y_k}{\sqrt{n}} + f''(U_k)\frac{Y_k^2}{2n} + f'''(U_k^*)\frac{Y_k^3}{6n^{\frac{3}{2}}}$$

$$f(V_k) - f(U_k) = f'(U_k)\frac{X_k}{\sqrt{n}} + f''(U_k)\frac{X_k^2}{2n} + f'''(U_k^{**})\frac{X_k^3}{6n^{\frac{3}{2}}}$$

In page number 33 , equation before section 17 , the power of n is written as 3/2 it should be 1/2.

In page number 33 , proof of theorem 41 from theorem 49 , second line , the summation is taken from 1 to N it should be 1 to n.

In page number 36 , after equation (15) , the inequalities are valid only when $\max_{k \le n} \sigma_{n,k}^2 \le \frac{1}{t^2}$ (the first inequality - page number 6, equation 1) , which can be done for fixed $t \neq 0$.

In page number 36 , section 18.2 , similar problem as that in section 16.2.

In page number 37 , equation 17 , left hand side is missing the function f.

In page number 37 , last but one equation , left hand side is missing the function f.

In page number 38 , first line , in the inequality C_f need not be written since it has already been absorbed into δ previously.

In page number 40 , first line , it is written $\psi_X(t) = \int_0^\infty \lambda e^{-\lambda x} e^{itx} dx = \frac{1}{\lambda - it}$, it should be $\psi_X(t) = \int_0^\infty \lambda e^{-\lambda x} e^{itx} dx = \frac{\lambda}{\lambda - it}$ ($\psi_X(0)$ should be 1). In the next line it is written $Y = X_1 + \cdots + X_n$ it should be $Y = X_1 + \cdots + X_\nu$.

In page number 41 , proof of theorem 61 , second set of equations is written as

$$\begin{aligned} \int_a^b f_\sigma(\alpha) d\alpha &= \int_a^b \int_{\mathbb{R}} \phi_{\frac{1}{\sigma}}(\alpha - x) d\mu(x) d\mu(x) \\ &= \int_{\mathbb{R}} \int_a^b \phi_{\frac{1}{\sigma}}(\alpha - x) d\alpha d\mu(x) \\ &= \int_{\mathbb{R}} \left(\Phi_{\frac{1}{\sigma}}(\alpha - a) - \Phi_{\frac{1}{\sigma}}(\alpha - b) \right) d\mu(x) \end{aligned}$$

it should be

$$\begin{aligned} \int_a^b f_\sigma(\alpha) d\alpha &= \int_a^b \int_{\mathbb{R}} \phi_{\frac{1}{\sigma}}(\alpha - x) d\mu(x) d\alpha \\ &= \int_{\mathbb{R}} \int_a^b \phi_{\frac{1}{\sigma}}(\alpha - x) d\alpha d\mu(x) \\ &= \int_{\mathbb{R}} \left(\Phi_{\frac{1}{\sigma}}(b - x) - \Phi_{\frac{1}{\sigma}}(a - x) \right) d\mu(x) \end{aligned}$$

In page number 42 , proof (2) , second paragraph , first line , it is written $C = \int |\hat{\mu}.$, the $|$ is missing.

In page number 42 , last line , it is written $\hat{nu}(t)$ it should be $\hat{\nu}(t)$ (the backslash is missing).

In page number 43 , proof of theorem 63 , (1) first line , it is written $\mu_n \to mu$ it should be $\mu_n \to \mu$ (the backslash is missing).

In page number 44 , proof of lemma 64 , in the third line a factor of 2 is missing , it should be

$$\int_{-\delta}^{\delta}(1-\hat{\mu}(t))dt = \int_{\mathbb{R}}\left(2\delta - \frac{2\sin(x\delta)}{x}\right)$$

$$= 2\delta\int_{\mathbb{R}}\left(1-\frac{\sin(x\delta)}{x\delta}\right)$$

and note that when $|x|\delta > 2$ we have $\frac{\sin(x\delta)}{x\delta} \leq \frac{1}{2}$. Also the final equation needs to be corrected , the 1/2 replaced by δ.