FIRST MID-TERM EXAMINATION
PROBABILITY AND STATISTICS
28 OCT 2013 – 9:30-10:40 AM

Instructions: *The duration of the test is* 70 *minutes. The maximum you can score is* 45*. The marks for each question is indicated in bold,* [**5**] *means that part carries* 5 *marks. Give all details but try to write succinctly. Please start your answer to each question (not each question part!) on a new page.*

1. [15] Suppose you have a random number generator which generates $\text{Exp}(1)$ random variables. Explain with justification how you would use it to sample numbers from the probability density

$$f(x) = \begin{cases} \frac{1}{\pi\sqrt{1-x^2}} & \text{if } -1 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

2. [10] Suppose $X \sim \text{Unif}[a,b]$ for some $a < b$. If $\mathbf{E}[X] = 3$ and $\text{Var}(X) = 6$, find the values of a and b.

3. [15] Let X and Y be independent random variables having $\text{Gamma}(\nu_1, 1)$ and $\text{Gamma}(\nu_2, 1)$ distributions, respectively. Find the distribution of $\frac{X}{X+Y}$.

4. A deck consists of cards labelled $1, 2, \ldots, N$. The deck is shuffled well. Let X be the label on the first card and let Y be the label on the second card.

(1) **[10]** Find the covariance of X and Y.
(2) **[5]** Let A be the event that X is at most 2 and let B be the event that Y is atmost 2. Find $\mathbf{P}(B \,|A)$.

Date: October 26, 2013.