

Thematic lectures

Andrei Lerner

Bar-Ilan University, Israel

**Title:** Sparse bounds and sharp weighted estimates

**Abstract:** The celebrated  $A_2$  theorem (proved by T. Hytonen in 2010) states that the weighted  $L^2$  norm of a Calderon-Zygmund operator is linear in terms of the  $A_2$  characteristic of the weight. Later, a relatively simple proof of this theorem was found that was based on a domination of Calderon-Zygmund operators by positive dyadic operators called the sparse operators. Since then, sparse operators (and corresponding sparse bounds) have become quite popular in Harmonic Analysis. In more than 30 works appeared in the last two years, sparse bounds have been established for a number of operators in various settings. In this series of talks I plan to give an overview of this subject and to discuss sparse bounds for Calderon-Zygmund operators and their commutators, and for rough singular integrals.

## Titles and Abstracts

**V. P. Anoop**

**NISER, Bhubaneswar, India**

**Title:** Hardy and fractional Hardy inequality for Dunkl-Laplacian

**Abstract:** In this talk we shall discuss about the Hardy inequality and fractional Hardy inequality related to Dunkl Laplacian. In the process we will obtain certain uncertainty principles in this context.

**Sayan Bagchi**

**Indian Statistical Institute, Kolkata**

**Title:** On Fourier multipliers on the Heisenberg groups

**Abstract:** In this talk we will discuss Fourier multipliers on the Heisenberg groups. We will modify the Hörmander type conditions given by Chin-Chen Lin for  $L^p$  boundedness of those operators. Then we will state a sharp weighted estimate for them.

**R. Daher**

**University of Hassan, Morocco**

**Title:** Titchmarsh theorems for Fourier transforms of Hölder-Lipschitz functions on compact homogeneous manifolds

**Abstract:** In this paper we extend classical Titchmarsh theorems on the Fourier transform of Hölder-Lipschitz functions to the setting of compact homogeneous manifolds. As an application, we derive a Fourier multiplier theorem for  $L^2$ -Hölder-Lipschitz spaces on compact Lie groups. We also derive conditions and a characterisation for Dini-Lipschitz classes on compact homogeneous manifolds in terms of the behaviour of their Fourier coefficients. (Based on a joint work with M. RUZHANSKY and J. DELGADO).

**T. C. Easwaran Namboodiri**

**Government Brennen College, Kerala**

**Title:** Canonical duals–Gabor frames versus wavelet frames

**Abstract:** From the perspective of duality, the classes of Gabor frames and wavelet frames have contrasting behaviour. Canonical duals of Gabor frames are again Gabor frames, whereas the canonical dual of a wavelet frame need not be a wavelet frame. Investigations around

this phenomenon lead us to constructions of several classes of wavelet frames whose canonical duals have the same wavelet frame structure (joint work with K. Parthasarathy).

**J. Faraut**

**Université Pierre et Marie Curie, France**

**Title: Projection of orbital measures for the action of a pseudo-unitary group.**

**Abstract:** Consider the action of the unitary group  $U(n)$  on the space  $Herm(n, \mathbb{C})$  of  $n \times n$  Hermitian matrices. The projection of an orbital measure on the subspace  $Herm(n-1, \mathbb{C})$  is described by a formula due to Baryshnikov (2001). Now we consider the action of the pseudo-unitary group  $U(p, q)$  on  $Herm(n, \mathbb{C})$ , ( $p+q=n$ ). For an orbit of convex type we prove an analogue of Baryshnikov's formula by using an evaluation of the Fourier-Laplace transform for the associated orbital measure due to Ben Säïd and Orsted (2005)

**G. B. Folland**

**University of Washington, Seattle, USA**

**Title: Remarks on Gabor systems**

**Abstract:** Given a function  $\phi$  in  $L^2(\mathbb{R})$  and positive numbers  $a$  and  $b$ , the *Gabor system* determined by  $(\phi, a, b)$  is the set of functions  $\{\phi_{mn} : m, n \in \mathbb{Z}\}$  where  $\phi_{mn}(t) = e^{2\pi i n b t} \phi(t - ma)$ . Question: when is this system a “frame” (a generalization of “basis”) for  $L^2(\mathbb{R})$ ? We will review some of the many results that have been obtained on aspects of this problem, focusing on one question that has been completely answered only recently: for which  $a, b$  does  $(\phi, a, b)$  give a frame when  $\phi$  is the characteristic function of an interval?

**Abhishek Ghosh**

**Indian Institute of Technology, Kanpur**

**Title: Sparse domination of multilinear Calderon-Zygmund operators on non-homogeneous spaces**

**Abstract:** In recent times, A. Lerner has obtained the pointwise domination of Calderon-Zygmund operators by Sparse operators and thus giving another proof of  $A_2$ -conjecture. In this talk we discuss about multilinear Calderon-Zygmund operators on non-homogeneous spaces and their pointwise domination by multilinear Sparse operators.

As a consequence we obtain sharp bounds for multilinear Calderon-Zygmund operators with respect to multilinear  $A_P$  weights. Joint work with Parasar Mohanty and Saurabh Srivastava.

**Guixiang Hong**

**Wuhan University, P. R. China**

**Title: Some progresses on noncommutative ergodic theory**

**Abstract:** In this talk, I shall first give a historical review on the ergodic theory associated to group actions. Then I shall present our recent works on noncommutative ergodic theory. Based on a joint work with Benben Liao and Simeng Wang.

**Qaiser Jahan**

**Indian Institute of Science, Bangalore**

**Title: Shearlet Coorbit Spaces**

**Abstract:** In this talk we investigate the traces of functions lying in certain subspaces of shearlet coorbit spaces with respect to more general lines. I begin my talk from the basic definitions of shearlet group, coorbit spaces and shearlet coorbit spaces. This is a joint work with S. Dahlke and G. Steidl.

**Jotsaroop Kaur**

**IISER Bhopal**

**Title: Unimodular bilinear multipliers on  $L^p$  spaces**

**Abstract:** We investigate boundedness properties of bilinear multiplier operators associated with unimodular functions of the form  $m(\xi, \eta) = e^{i\phi(\xi-\eta)}$ . This is based on a joint work with Saurabh Srivastava.

**Bernhard Krötz**

**University of Paderborn, Germany**

**Title: The discrete spectrum of a real spherical space**

A homogeneous space  $Z = G/H$  attached to a real reductive group  $G$  and closed subgroup  $H$  is termed real spherical provided the natural left action of a minimal parabolic subgroup  $P = MAN$  of  $G$  on  $Z$  admits open orbits. Every symmetric space is real spherical space and recently we obtained a complete classification of all real spherical

spaces with  $H$  reductive in  $G$ . This talk will be about the discrete spectrum of  $L^2(Z)$ . We will explain that the infinitesimal characters of the discrete spectrum all lie in certain lattice. In particular the discrete spectrum features a spectral gap theorem: the real parts of the parameters are bounded away from  $\rho$ . This in turn allows us to implement the Bernstein morphisms which give an isospectral decomposition of  $L^2(Z)$  in terms of the relative discrete series of the natural satellites of  $Z$  obtained via a wonderful compactification of  $Z$ . The results were obtained in joint work with Patrick Delorme, Friedrich Knop, Job Kuit, Eric Opdam and Henrik Schlichtkrull in two different projects.

**Lakshmi Lavanya**

**IISER Tirupati, India**

**Title: On Fourier twin algebras in locally compact Abelian groups.**

**Abstract:** In this talk, we will discuss a characterisation of the Fourier transform on the space of Schwartz-Bruhat functions and the Feichtinger algebra on locally compact Abelian groups. The result states that any appropriately additive bijection from the Schwartz-Bruhat space (resp. the Feichtinger algebra) of a locally compact Abelian group onto the Schwartz-Bruhat space (resp. the Feichtinger algebra) of its dual, which interchanges convolution and pointwise products, is essentially the Fourier transform. We obtain this as a consequence of a similar result for the more general class of algebras, which we call Fourier twin algebras

**Elijah Liflyand**

**Bar-Ilan University, Israel**

**Title: Hausdorff operators in  $H^p$  spaces,  $0 < p < 1$**

**Abstract:** For the theory of Hardy spaces  $H^p$ ,  $0 < p < 1$ , the Hausdorff operators turn out to be a very effective testing area, in dimension one and especially in several dimensions. In contrast to the study of the Hausdorff operators in  $L^p$ ,  $1 \leq p \leq \infty$ , and in the Hardy space  $H^1$ , the study of these operators in the Hardy spaces  $H^p$  with  $p < 1$  holds a specific place and there are very few results on this topic. For the case of one dimension, after the work of Kanjin and Miyachi, more or less final results were given in our joint paper with Miyachi. The results differ from those for  $L^p$ ,  $1 \leq p \leq \infty$ , and  $H^1$ , since they involve smoothness conditions on the averaging function, which seem unusual

but unavoidable. To explain them will be the main purpose of the talk. This leads to better understanding even more specific difficulties in our multidimensional joint work with Akihiko Miyachi.

**Ramesh Manna**  
TIFR, Bangalore

**Title: On local smoothing of Fourier integral operators and its applications**

**Abstract:** In this talk, we discuss the "local smoothing" phenomenon for Fourier integral operators with amplitude function  $a(x, t, \xi)$  belonging to  $S^m$ , the symbol class of order  $m$  less or equal to 0. We give an overview of the local smoothing results which have been proven to date. Such operators arise in solving the wave equation and also in the study of spherical maximal operators. Finally, we give a different proof of the  $L^p$ -boundedness of Bourgain's circular maximal operator on  $L^p(\mathbb{R}^2)$  for  $p > 2$  with the help of the above local smoothing estimate.

**G. Pisier**  
Texas A&M University, USA

**Title: Sidon sets in discrete groups**

**Abstract:** We will recall some of the classical theory of Sidon sets of characters on compact groups (Abelian or not). This was recently extended to bounded orthonormal systems, following recent work by Bourgain and Lewko, and by the author, both currently available on arxiv. We will then give several more recent extensions to discrete groups e.g. free groups. The latter case is closely connected to operator space theory.

**Radha Ramakrishnan**  
Indian Institute of Technology, Chennai

**Title: Frames and shift invariant spaces on the Heisenberg group**

**Abstract:** It is well known that the frames and Riesz basis of system of integer translates for the principal shift invariant space on the real line can be characterized in terms of the Fourier transform with integer periodization. This integer periodization of Fourier transform leads to a positive weight function  $W$ . It is again well known that the system of integer translates form a Schauder basis for the principal shift invariant

space if and only if the weight function  $W$  belongs to the Muckenhoupt  $A_2$  class. The aim of the talk is to discuss these problems for the twisted shift invariant space on  $\mathbb{C}^n$  and shift invariant space on the Heisenberg group.

**Samya Kumar Ray**

**Indian Institute of Technology, Kanpur**

**Title: Joint Functional Calculus of Ritt Operators**

**Abstract:** It is well known that for arbitrary  $n$ -tuple of commuting contractions von Neumann inequality and dilation theorems do not hold. In this talk we will see that if we restrict ourselves to a subclass of commuting tuple of Ritt operators we get some positive results in this direction. As an application we can provide an affirmative answer to the joint similarity problem for these type of operators. Our proofs rely on boundedness of Littlewood-Paley type square functions and certain transfer principles. This is a joint work with Parasar Mohanty.

**Fulvio Ricci**

**SNS, Italy**

**Title: Functional calculus on sublaplacians and Gelfand pairs**

**Abstract:** It follows from the works of Hulanicki-Stein and Alexopoulos that, given a Schwartz multiplier  $m$  on the line and a left-invariant sublaplacian  $L$  on a Lie group  $G$  with polynomial volume growth, the operator  $m(L)$  is given by right convolution with a Schwartz kernel  $K_m$ .

The converse statement, i.e.,  $K_m \in \mathcal{S}(G) \implies m$  extends to a function in  $\mathcal{S}(\mathbb{R})$ , turns out to hold true in special cases (e.g., if  $L$  is homogeneous on a stratified group). In this talk we present recent progress in the direction of generalizing this result and extensions to joint functional calculus for commuting families of left-invariant operators. This brings in Gelfand pairs and properties of their spherical transform.

**Luz Roncal**

**Basque center for applied mathematics, Spain**

**Title: Hardy inequalities for fractional sublaplacians**

**Abstract:** We study Hardy inequalities for the conformally invariant fractional powers of the sublaplacian, denoted by  $\mathcal{L}_s$ , on the Heisenberg group. Our approach uses trace Hardy inequalities, for which we need to find solutions of an extension problem associated to sublaplacians. This leads us to revisit the recent works on the topic by Frank-González-Monticelli-Tan and Möllers-Ørsted-Zhang. The present work has been accomplished in collaboration with S. Thangavelu.

**Rajesh Kumar Singh**

**IIT Kanpur, India**

**Title: A sharp form of the Marcinkiewicz Interpolation Theorem for Orlicz spaces**

**Abstract:** An important special case of the classical Marcinkiewicz interpolation theorem asserts that a quasilinear operator defined on a space of simple functions on a measure space  $(X, \mu)$  and taking values in the space of measurable functions on measure space  $(Y, \nu)$ , which is of weak-type  $(p_0, p_0)$  and  $(p_1, p_1)$ , is bounded from  $L_p(X, \mu)$  to  $L_p(Y, \nu)$  for  $p_0 < p < p_1$ . This theorem has been extended to include various important generalizations. We, in particular, focus on the work of Zygmund-Strömberg (1956, 1975), A. Cianchi (1998, 1999) and R. Kerman *et al.* (2014), that extends this theorem to a special class of Banach function spaces, namely, Orlicz spaces. In this talk, we build on these authors' work and consider a version of the Marcinkiewicz interpolation theorem for the class,  $W((p_0, r_0), (p_1, r_1); \mu, \nu)$ , of quasilinear operators that map Lorentz space  $L_{p_0, r_0}(X, \mu)$  into  $L_{p_0, \infty}(Y, \nu)$  and  $L_{p_1, r_1}(X, \mu)$  into  $L_{p_1, \infty}(Y, \nu)$ ,  $1 < p_0 < p_1 < \infty$ ,  $1 \leq r_0, r_1 < \infty$ . We give necessary and sufficient conditions for such operators to be bounded from Orlicz space,  $L_{\Phi_2}(X, \mu)$ , to Orlicz space,  $L_{\Phi_1}(Y, \nu)$ .

A special case of the above problem will be presented in the talk. Thus, we would focus on quasilinear operators from  $L_{p_0, r_0}(X, \mu)$  into  $L_{p_0, \infty}(Y, \nu)$  and  $L_\infty(X, \mu)$  into  $L_\infty(Y, \nu)$ . We give a brief sketch of the ideas involved in the proof which has important ingredients from the theory of interpolation, Hardy inequalities for weights, duality principle of Sawyer and the work of Stepanov on integral operators.



M. A. Sofi

Kashmir University, Srinagar

**Title:** Banach spaces in the Lipschitz category

**Abstract:** Over the past several decades of research in Banach space theory, a novel viewpoint has emerged that it is possible to capture the linear structure of a Banach space in terms of the underlying metric structure which is induced by the norm on the given space. With the class of Lipschitz maps now playing the role of morphisms in this new category, it turns out that the linear structure of a Banach space is closely tied to these nonlinear objects which encode a lot of information on the linear structure of the Banach space in question. The first signs of this close chemistry between these disparate structures of a Banach space were provided by a famous theorem of Enflo which asserts that a Banach space is linearly homeomorphic to a Hilbert space as soon as it is Lipschitz (even uniformly) homeomorphic to it.

The purpose of this talk is to draw attention to the interplay of these structures and to their implications on the linear structure of the underlying Banach space. We shall do so in the context of extendability of Lipschitz maps in Banach spaces and the related issue involving the question of the extension being chosen in a linear and continuous manner. Whereas such a choice is always available in Hilbert space, it turns out that Hilbert spaces are (isomorphically) the only Banach spaces where such a choice is possible.

Krzysztof Stempak

Wroclaw University of Science and Technology, Poland

**Title:** Reflection principle for the Dirichlet and Neumann Laplacians on open reflection invariant sets

**Abstract:** For an open symmetric in a hyperplane subset  $\Omega$  of  $\mathbb{R}^d$  with a positive part  $\Omega_+$ , we consider the Neumann/Dirichlet Laplacians  $-\Delta_{N/D,\Omega}$  and  $-\Delta_{N/D,\Omega_+}$ . Given a bounded Borel function  $\Phi$  on  $(0, \infty)$  we apply the spectral functional calculus and consider the pairs of operators  $\Phi(-\Delta_{N,\Omega})$  and  $\Phi(-\Delta_{N,\Omega_+})$ , or  $\Phi(-\Delta_{D,\Omega})$  and  $\Phi(-\Delta_{D,\Omega_+})$ . We prove relations between the integral kernels for the operators in these pairs, which in particular cases of  $\Omega_+ = \mathbb{R}^{d-1} \times (0, \infty)$  and  $\Phi_t(x) = \exp(-tx)$ ,  $x > 0$ , were known as reflection principles for the Neumann/Dirichlet heat kernels.

M Sundari

Chennai Mathematical Institute, Chennai

**Title:** Analogues of Benedicks theorem on two step nilpotent Lie groups

**Abstract:** We shall look at various analogues of Benedicks theorem. This is joint work with Suparna Sen.

Yuan Xu

University of Oregon, USA

**Title:** Approximation and Orthogonality in Sobolev Spaces

**Abstract:** It is well known that polynomials of best approximation in an  $L^2$  space are partial sums of the Fourier orthogonal expansions. If we were to approximate functions and their derivatives simultaneously on a domain in  $R^d$  (as desired in spectral method), we would need to consider orthogonal expansions in a Sobolev space, for which the orthogonality is defined with respect to an inner product that contains derivatives. Since multiplication operators are no longer self-adjoint under such an inner product, the orthogonality is much hard to understand and analyze. We explain recent results and development in this talk.