Joint functional calculus of Ritt operators

Samya Kumar Ray
IIT Kanpur
samya@iitk.ac.in

Joint work with Prof. P Mohanty

December 20, 2017

1 von Neumann inequality

von Neumann inequality says that

1von Neumann inequality

von Neumann inequality says that

Given any contraction T on a Hilbert space $\mathcal{H}$ and a polynomial P in single variable

$$\|P(T)\|_{\mathcal{H}\to\mathcal{H}} \leq \|P\|_{\infty,\mathbb{D}}.$$

1 von Neumann inequality

von Neumann inequality says that

Given any contraction T on a Hilbert space $\mathcal{H}$ and a polynomial P in single variable
$$\|P(T)\|_{\mathcal{H}\to\mathcal{H}} \leq \|P\|_{\infty,\mathbb{D}}.$$

As a consequence we immediately obtain a *rational functional calculus*.

1 von Neumann inequality

von Neumann inequality says that

Given any contraction T on a Hilbert space $\mathcal{H}$ and a polynomial P in single variable

$$\|P(T)\|_{\mathcal{H}\to\mathcal{H}} \leq \|P\|_{\infty,\mathbb{D}}.$$

As a consequence we immediately obtain a *rational functional calculus*.

Several attempts were made to generalize von Neumann inequality. These are

[1]von Neumann inequality

von Neumann inequality says that

Given any contraction T on a Hilbert space $\mathcal{H}$ and a polynomial P in single variable

$$\|P(T)\|_{\mathcal{H}\to\mathcal{H}} \leq \|P\|_{\infty,\mathbb{D}}.$$

As a consequence we immediately obtain a *rational functional calculus*.

Several attempts were made to generalize von Neumann inequality. These are

- A generalization in two variables due to dilation theorem of Ando.

[1]von Neumann inequality

von Neumann inequality says that

Given any contraction T on a Hilbert space $\mathcal{H}$ and a polynomial P in single variable

$$\|P(T)\|_{\mathcal{H}\to\mathcal{H}} \leq \|P\|_{\infty,\mathbb{D}}.$$

As a consequence we immediately obtain a *rational functional calculus*.

Several attempts were made to generalize von Neumann inequality. These are

- A generalization in two variables due to dilation theorem of Ando.
- Replacing Hilbert space by general Banach space. This involves the so called Matsaev's conjecture. (**Upcoming slide.**)

$_1$von Neumann inequality

von Neumann inequality says that

Given any contraction T on a Hilbert space $\mathcal{H}$ and a polynomial P in single variable

$$\|P(T)\|_{\mathcal{H}\to\mathcal{H}} \leq \|P\|_{\infty,\mathbb{D}}.$$

As a consequence we immediately obtain a *rational functional calculus*.

Several attempts were made to generalize von Neumann inequality. These are

- A generalization in two variables due to dilation theorem of Ando.
- Replacing Hilbert space by general Banach space. This involves the so called Matsaev's conjecture. (**Upcoming slide.**)

However, Varopoulos and Kaijser gave an explicit counterexample for three commuting contractions.

2 Loose dilation

We need the following definition.

2 Loose dilation

We need the following definition.

Let $1 < p \neq 2 < \infty$. $\mathbf{T} = (T_1, \ldots, T_n)$ is commuting tuple of bounded operators on $L^p(\Omega)$.

Loose dilation

We need the following definition.

Let $1 < p \neq 2 < \infty$. $\mathbf{T} = (T_1, \ldots, T_n)$ is commuting tuple of bounded operators on $L^p(\Omega)$. We say $\mathbf{T}$ admits a joint isometric loose dilation if there is a measure space Ω' and a commuting tuple of onto isometries $\mathbf{U} = (U_1, \ldots, U_n)$, on $L^p(\Omega')$

2 Loose dilation

We need the following definition.

Let $1 < p \neq 2 < \infty$. $\mathbf{T} = (T_1, \ldots, T_n)$ is commuting tuple of bounded operators on $L^p(\Omega)$. We say $\mathbf{T}$ admits a joint isometric loose dilation if there is a measure space Ω' and a commuting tuple of onto isometries $\mathbf{U} = (U_1, \ldots, U_n)$, on $L^p(\Omega')$ together with bounded operators $\mathcal{Q} : L^p(\Omega') \to L^p(\Omega)$ and $\mathcal{J} : L^p(\Omega) \to L^p(\Omega')$, such that

$$T_1^{i_1} \cdots T_n^{i_n} = \mathcal{Q} U_1^{i_1} \cdots U_n^{i_n} \mathcal{J}$$

for all $i_1, \ldots, i_n \in \mathbb{N}_0$.

2 Loose dilation

We need the following definition.

Let $1 < p \neq 2 < \infty$. $\mathbf{T} = (T_1, \ldots, T_n)$ is commuting tuple of bounded operators on $L^p(\Omega)$. We say $\mathbf{T}$ admits a joint isometric loose dilation if there is a measure space Ω' and a commuting tuple of onto isometries $\mathbf{U} = (U_1, \ldots, U_n)$, on $L^p(\Omega')$ together with bounded operators $\mathcal{Q} : L^p(\Omega') \to L^p(\Omega)$ and $\mathcal{J} : L^p(\Omega) \to L^p(\Omega')$, such that

$$T_1^{i_1} \cdots T_n^{i_n} = \mathcal{Q} U_1^{i_1} \cdots U_n^{i_n} \mathcal{J}$$

for all $i_1, \ldots, i_n \in \mathbb{N}_0$.

In other words following diagram commutes

$$\begin{array}{ccc} L^p(\Omega) & \xrightarrow{T_1^{i_1} \cdots T_n^{i_n}} & L^p(\Omega) \\ \Big\downarrow \mathcal{J} & & \Big\uparrow \mathcal{Q} \\ L^p(\Omega') & \xrightarrow{U_1^{i_1} \cdots U_n^{i_n}} & L^p(\Omega') \end{array} .$$

3 Loose dilation continued...

Some examples of dilations are:

(1) (Nagy-Foias) Contractions on Hilbert spaces admit strict dilation.

Loose dilation continued...

Some examples of dilations are:

(1) (Nagy-Foias) Contractions on Hilbert spaces admit strict dilation.

(2) (Ando, 1963) Any commuting couple of contractions on a Hilbert space admits a joint isometric strict dilation.

Loose dilation continued...

Some examples of dilations are:

(1) (Nagy-Foias) Contractions on Hilbert spaces admit strict dilation.

(2) (Ando, 1963) Any commuting couple of contractions on a Hilbert space admits a joint isometric strict dilation.

(3) (Akcoglu, Coifman-Weiss-Rochberg, 1977-78) Sub-positive contractions on L^p-spaces, $1 < p \neq 2 < \infty$ admit strict dilation. (**What happens in multivariable?)**

Loose dilation continued...

Some examples of dilations are:

(1) (Nagy-Foias) Contractions on Hilbert spaces admit strict dilation.

(2) (Ando, 1963) Any commuting couple of contractions on a Hilbert space admits a joint isometric strict dilation.

(3) (Akcoglu, Coifman-Weiss-Rochberg, 1977-78) Sub-positive contractions on L^p-spaces, $1 < p \neq 2 < \infty$ admit strict dilation. (**What happens in multivariable?)**

(4) (Le Merdy, Arhancet and Fackler, 2014, 2017) Ritt operators on an L^p-space which satisfies a bounded H^∞-functional calculus admits isometric loose dilation for $1 < p < \infty$. (**Next slide.)**

$_3$Loose dilation continued...

Some examples of dilations are:

(1) (Nagy-Foias) Contractions on Hilbert spaces admit strict dilation.

(2) (Ando, 1963) Any commuting couple of contractions on a Hilbert space admits a joint isometric strict dilation.

(3) (Akcoglu, Coifman-Weiss-Rochberg, 1977-78) Sub-positive contractions on L^p-spaces, $1 < p \neq 2 < \infty$ admit strict dilation. (**What happens in multivariable?)**

(4) (Le Merdy, Arhancet and Fackler, 2014, 2017) Ritt operators on an L^p-space which satisfies a bounded H^∞-functional calculus admits isometric loose dilation for $1 < p < \infty$. (**Next slide.)**

- Loose dilation $\implies$ von Neumann type inequality, e.g., for any sub-positive contraction T on an L^p-space, $1 < p < \infty$ the Matsaev's conjecture holds i.e.

[3] Loose dilation continued...

Some examples of dilations are:

(1) (Nagy-Foias) Contractions on Hilbert spaces admit strict dilation.

(2) (Ando, 1963) Any commuting couple of contractions on a Hilbert space admits a joint isometric strict dilation.

(3) (Akcoglu, Coifman-Weiss-Rochberg, 1977-78) Sub-positive contractions on L^p-spaces, $1 < p \neq 2 < \infty$ admit strict dilation. (**What happens in multivariable?)**

(4) (Le Merdy, Arhancet and Fackler, 2014, 2017) Ritt operators on an L^p-space which satisfies a bounded H^∞-functional calculus admits isometric loose dilation for $1 < p < \infty$. (**Next slide.)**

- Loose dilation $\implies$ von Neumann type inequality, e.g., for any sub-positive contraction T on an L^p-space, $1 < p < \infty$ the Matsaev's conjecture holds i.e.

$$\|P(T)\|_{L^p \to L^p} \leq \|P(R)\|_{\ell_p(\mathbb{N}) \to \ell_p(\mathbb{N})}.$$

Loose dilation continued...

Some examples of dilations are:

(1) (Nagy-Foias) Contractions on Hilbert spaces admit strict dilation.

(2) (Ando, 1963) Any commuting couple of contractions on a Hilbert space admits a joint isometric strict dilation.

(3) (Akcoglu, Coifman-Weiss-Rochberg, 1977-78) Sub-positive contractions on L^p-spaces, $1 < p \neq 2 < \infty$ admit strict dilation. (**What happens in multivariable?)**

(4) (Le Merdy, Arhancet and Fackler, 2014, 2017) Ritt operators on an L^p-space which satisfies a bounded H^∞-functional calculus admits isometric loose dilation for $1 < p < \infty$. (**Next slide.)**

- Loose dilation $\implies$ von Neumann type inequality, e.g., for any sub-positive contraction T on an L^p-space, $1 < p < \infty$ the Matsaev's conjecture holds i.e.

$$\|P(T)\|_{L^p \to L^p} \leq \|P(R)\|_{\ell_p(\mathbb{N}) \to \ell_p(\mathbb{N})}.$$

- In above R is the shift operator on $\ell_p(\mathbb{N})$.

Ritt operator

Ritt operator: For $\gamma \in (0, \frac{\pi}{2})$, let $\mathcal{B}_\gamma$ (**Stolz** domain of angle γ) be the interior of the convex hull of 1 and the disc $D(0, \sin\gamma)$. An operator $T : X \to X$ is said to be a Ritt operator of type $\alpha \in (0, \frac{\pi}{2})$ if

(1) $\sigma(T) \subseteq \overline{\mathcal{B}_\alpha}$.

$_4$ Ritt operator

Ritt operator: For $\gamma \in (0, \frac{\pi}{2})$, let $\mathcal{B}_\gamma$ (**Stolz** domain of angle γ) be the interior of the convex hull of 1 and the disc $D(0, \sin\gamma)$. An operator $T : X \to X$ is said to be a Ritt operator of type $\alpha \in (0, \frac{\pi}{2})$ if

(1) $\sigma(T) \subseteq \overline{\mathcal{B}_\alpha}$.

(2) For any $\beta \in (\alpha, \frac{\pi}{2})$, the set $\{(1-\lambda)R(\lambda, T) : \lambda \in \mathbb{C} \setminus \overline{\mathcal{B}_\beta}\}$ is bounded.

Ritt operator

Ritt operator: For $\gamma \in (0, \frac{\pi}{2})$, let $\mathcal{B}_\gamma$ (**Stolz** domain of angle γ) be the interior of the convex hull of 1 and the disc $D(0, \sin\gamma)$. An operator $T : X \to X$ is said to be a Ritt operator of type $\alpha \in (0, \frac{\pi}{2})$ if

(1) $\sigma(T) \subseteq \overline{\mathcal{B}_\alpha}$.

(2) For any $\beta \in (\alpha, \frac{\pi}{2})$, the set $\{(1-\lambda)R(\lambda, T) : \lambda \in \mathbb{C} \setminus \overline{\mathcal{B}_\beta}\}$ is bounded.

- Following characterization of Ritt operators were obtained by a series of work of Lyubich, Nagy-Zemanek and Nevanlinna.

Ritt operator

Ritt operator: For $\gamma \in (0, \frac{\pi}{2})$, let $\mathcal{B}_\gamma$ (**Stolz** domain of angle γ) be the interior of the convex hull of 1 and the disc $D(0, \sin\gamma)$. An operator $T : X \to X$ is said to be a Ritt operator of type $\alpha \in (0, \frac{\pi}{2})$ if

(1) $\sigma(T) \subseteq \overline{\mathcal{B}_\alpha}$.

(2) For any $\beta \in (\alpha, \frac{\pi}{2})$, the set $\{(1-\lambda)R(\lambda, T) : \lambda \in \mathbb{C} \setminus \overline{\mathcal{B}_\beta}\}$ is bounded.

- Following characterization of Ritt operators were obtained by a series of work of Lyubich, Nagy-Zemanek and Nevanlinna.

Theorem *Let $T : X \to X$ be an operator. Then T is Ritt iff*

Ritt operator

Ritt operator: For $\gamma \in (0, \frac{\pi}{2})$, let $\mathcal{B}_\gamma$ (**Stolz** domain of angle γ) be the interior of the convex hull of 1 and the disc $D(0, \sin\gamma)$. An operator $T : X \to X$ is said to be a Ritt operator of type $\alpha \in (0, \frac{\pi}{2})$ if

(1) $\sigma(T) \subseteq \overline{\mathcal{B}_\alpha}$.

(2) For any $\beta \in (\alpha, \frac{\pi}{2})$, the set $\{(1-\lambda)R(\lambda, T) : \lambda \in \mathbb{C} \setminus \overline{\mathcal{B}_\beta}\}$ is bounded.

- Following characterization of Ritt operators were obtained by a series of work of Lyubich, Nagy-Zemanek and Nevanlinna.

Theorem *Let $T : X \to X$ be an operator. Then T is Ritt iff*

(1) T is power bounded.

Ritt operator

Ritt operator: For $\gamma \in (0, \frac{\pi}{2})$, let $\mathcal{B}_\gamma$ (**Stolz** domain of angle γ) be the interior of the convex hull of 1 and the disc $D(0, \sin\gamma)$. An operator $T : X \to X$ is said to be a Ritt operator of type $\alpha \in (0, \frac{\pi}{2})$ if

(1) $\sigma(T) \subseteq \overline{\mathcal{B}_\alpha}$.

(2) For any $\beta \in (\alpha, \frac{\pi}{2})$, the set $\{(1-\lambda)R(\lambda, T) : \lambda \in \mathbb{C} \setminus \overline{\mathcal{B}_\beta}\}$ is bounded.

- Following characterization of Ritt operators were obtained by a series of work of Lyubich, Nagy-Zemanek and Nevanlinna.

Theorem *Let $T : X \to X$ be an operator. Then T is Ritt iff*

(1) T is power bounded.

(2) The set $\{n(T^n - T^{n-1}) : n \geq 1\}$ is bounded.

Ritt operator

Ritt operator: For $\gamma \in (0, \frac{\pi}{2})$, let $\mathcal{B}_\gamma$ (**Stolz** domain of angle γ) be the interior of the convex hull of 1 and the disc $D(0, \sin\gamma)$. An operator $T : X \to X$ is said to be a Ritt operator of type $\alpha \in (0, \frac{\pi}{2})$ if

(1) $\sigma(T) \subseteq \overline{\mathcal{B}_\alpha}$.

(2) For any $\beta \in (\alpha, \frac{\pi}{2})$, the set $\{(1-\lambda)R(\lambda, T) : \lambda \in \mathbb{C} \setminus \overline{\mathcal{B}_\beta}\}$ is bounded.

- Following characterization of Ritt operators were obtained by a series of work of Lyubich, Nagy-Zemanek and Nevanlinna.

Theorem *Let $T : X \to X$ be an operator. Then T is Ritt iff*

(1) T is power bounded.

(2) The set $\{n(T^n - T^{n-1}) : n \geq 1\}$ is bounded.

- Ritt operators are discrete analouge of **sectorial** operators. (**Upcoming slide.**)

Ritt operator

Ritt operator: For $\gamma \in (0, \frac{\pi}{2})$, let $\mathcal{B}_\gamma$ (**Stolz** domain of angle γ) be the interior of the convex hull of 1 and the disc $D(0, \sin\gamma)$. An operator $T : X \to X$ is said to be a Ritt operator of type $\alpha \in (0, \frac{\pi}{2})$ if

(1) $\sigma(T) \subseteq \overline{\mathcal{B}_\alpha}$.

(2) For any $\beta \in (\alpha, \frac{\pi}{2})$, the set $\{(1-\lambda)R(\lambda, T) : \lambda \in \mathbb{C} \setminus \overline{\mathcal{B}_\beta}\}$ is bounded.

- Following characterization of Ritt operators were obtained by a series of work of Lyubich, Nagy-Zemanek and Nevanlinna.

Theorem *Let $T : X \to X$ be an operator. Then T is Ritt iff*

(1) T is power bounded.

(2) The set $\{n(T^n - T^{n-1}) : n \geq 1\}$ is bounded.

- Ritt operators are discrete analouge of **sectorial** operators. (**Upcoming slide.)**
- For any $f \in L^1$ with $\|f\|_{L^1} < 1$ the map $f \mapsto f * g$ is a Ritt operator on L^p-space.

5 Joint functional calculus for Ritt operators.

- Let $\gamma_i \in (0, \frac{\pi}{2}),\ 1 \leq i \leq n$. Denote $H_0^\infty\big(\prod_{i=1}^n \mathcal{B}_{\gamma_i}\big)$ to be all bounded holomorphic functions $\phi : \prod_{i=1}^n \mathcal{B}_{\gamma_i} \to \mathbb{C}$ such that

5 Joint functional calculus for Ritt operators.

- Let $\gamma_i \in (0, \frac{\pi}{2}),\ 1 \leq i \leq n$. Denote $H_0^\infty\big(\prod_{i=1}^n \mathcal{B}_{\gamma_i}\big)$ to be all bounded holomorphic functions $\phi : \prod_{i=1}^n \mathcal{B}_{\gamma_i} \to \mathbb{C}$ such that

$$|\phi(\lambda_1, \dots, \lambda_n)| \leq c \prod_{i=1}^n |1 - \lambda_i|^s.$$

Joint functional calculus for Ritt operators.

- Let $\gamma_i \in (0, \frac{\pi}{2}),\ 1 \leq i \leq n$. Denote $H_0^\infty\big(\prod_{i=1}^n \mathcal{B}_{\gamma_i}\big)$ to be all bounded holomorphic functions $\phi : \prod_{i=1}^n \mathcal{B}_{\gamma_i} \to \mathbb{C}$ such that

$$|\phi(\lambda_1, \ldots, \lambda_n)| \leq c \prod_{i=1}^n |1 - \lambda_i|^s.$$

- Let $\Gamma_{\mathcal{B}_\beta}$ denote the boundary of $\mathcal{B}_\beta$ oriented counterclockwise.

Joint functional calculus for Ritt operators.

- Let $\gamma_i \in (0, \frac{\pi}{2}),\ 1 \leq i \leq n$. Denote $H_0^\infty\big(\prod_{i=1}^n \mathcal{B}_{\gamma_i}\big)$ to be all bounded holomorphic functions $\phi : \prod_{i=1}^n \mathcal{B}_{\gamma_i} \to \mathbb{C}$ such that

$$|\phi(\lambda_1, \ldots, \lambda_n)| \leq c \prod_{i=1}^{n} |1 - \lambda_i|^s.$$

- Let $\Gamma_{\mathcal{B}_\beta}$ denote the boundary of $\mathcal{B}_\beta$ oriented counterclockwise.
- Let $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple such that each T_i is Ritt operator of type $\alpha_i \in (0, \frac{\pi}{2})$ for $1 \leq i \leq n$.

Joint functional calculus for Ritt operators.

- Let $\gamma_i \in (0, \frac{\pi}{2}),\ 1 \leq i \leq n$. Denote $H_0^\infty\big(\prod_{i=1}^n \mathcal{B}_{\gamma_i}\big)$ to be all bounded holomorphic functions $\phi : \prod_{i=1}^n \mathcal{B}_{\gamma_i} \to \mathbb{C}$ such that

$$|\phi(\lambda_1, \ldots, \lambda_n)| \leq c \prod_{i=1}^n |1 - \lambda_i|^s.$$

- Let $\Gamma_{\mathcal{B}_\beta}$ denote the boundary of $\mathcal{B}_\beta$ oriented counterclockwise.
- Let $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple such that each T_i is Ritt operator of type $\alpha_i \in (0, \frac{\pi}{2})$ for $1 \leq i \leq n$.
- For $\beta_i \in (\alpha_i, \gamma_i)$, $1 \leq i \leq n$. We define

Joint functional calculus for Ritt operators.

- Let $\gamma_i \in (0, \frac{\pi}{2}),\ 1 \leq i \leq n$. Denote $H_0^\infty\big(\prod_{i=1}^n \mathcal{B}_{\gamma_i}\big)$ to be all bounded holomorphic functions $\phi : \prod_{i=1}^n \mathcal{B}_{\gamma_i} \to \mathbb{C}$ such that

$$|\phi(\lambda_1, \ldots, \lambda_n)| \leq c \prod_{i=1}^n |1 - \lambda_i|^s.$$

- Let $\Gamma_{\mathcal{B}_\beta}$ denote the boundary of $\mathcal{B}_\beta$ oriented counterclockwise.
- Let $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple such that each T_i is Ritt operator of type $\alpha_i \in (0, \frac{\pi}{2})$ for $1 \leq i \leq n$.
- For $\beta_i \in (\alpha_i, \gamma_i)$, $1 \leq i \leq n$. We define

$$\phi(\mathbf{T}) := \Big(\frac{1}{2\pi i}\Big)^n \int_{\prod_{i=1}^n \Gamma_{\mathcal{B}_{\beta_i}}} \phi(\lambda_1, \ldots, \lambda_n) \prod_{i=1}^n R(\lambda_i, A_i) d\lambda_i.$$

Joint functional calculus for Ritt operators.

- Let $\gamma_i \in (0, \frac{\pi}{2}),\ 1 \leq i \leq n$. Denote $H_0^\infty\big(\prod_{i=1}^n \mathcal{B}_{\gamma_i}\big)$ to be all bounded holomorphic functions $\phi : \prod_{i=1}^n \mathcal{B}_{\gamma_i} \to \mathbb{C}$ such that

$$|\phi(\lambda_1, \ldots, \lambda_n)| \leq c \prod_{i=1}^n |1 - \lambda_i|^s.$$

- Let $\Gamma_{\mathcal{B}_\beta}$ denote the boundary of $\mathcal{B}_\beta$ oriented counterclockwise.
- Let $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple such that each T_i is Ritt operator of type $\alpha_i \in (0, \frac{\pi}{2})$ for $1 \leq i \leq n$.
- For $\beta_i \in (\alpha_i, \gamma_i)$, $1 \leq i \leq n$. We define

$$\phi(\mathbf{T}) := \left(\frac{1}{2\pi i}\right)^n \int_{\prod_{i=1}^n \Gamma_{\mathcal{B}_{\beta_i}}} \phi(\lambda_1, \ldots, \lambda_n) \prod_{i=1}^n R(\lambda_i, A_i) d\lambda_i.$$

- We say $\mathbf{T}$ admits a joint **bounded** H^∞-functional calculus (in short j.b.f.c.) if the homomorphism $\phi \mapsto \phi(\mathbf{T})$ is bounded.

6 R-boundedness

- Denote the probability space $\Omega_0 = \{\pm 1\}^{\mathbb{Z}}$.

6 R-boundedness

- Denote the probability space $\Omega_0 = \{\pm 1\}^{\mathbb{Z}}$. Denote the k-th **Rademacher** function to be $\epsilon_k(\omega) = \omega_k$.

R-boundedness

- Denote the probability space $\Omega_0 = \{\pm 1\}^{\mathbb{Z}}$. Denote the k-th **Rademacher** function to be $\epsilon_k(\omega) = \omega_k$.
- For $1 \leq p < \infty$ we denote the Banach space $\mathrm{Rad}_p(X) \subseteq L^p(\Omega_0, X)$ to be the closure of the set span$\{\epsilon_k \otimes x_k : k \in \mathbb{Z}, x_k \in X\}$ in the Bochner space $L^p(\Omega_0, X)$.

R-boundedness

- Denote the probability space $\Omega_0 = \{\pm 1\}^{\mathbb{Z}}$. Denote the k-th **Rademacher** function to be $\epsilon_k(\omega) = \omega_k$.
- For $1 \leq p < \infty$ we denote the Banach space $\mathrm{Rad}_p(X) \subseteq L^p(\Omega_0, X)$ to be the closure of the set $\mathrm{span}\{\epsilon_k \otimes x_k : k \in \mathbb{Z}, x_k \in X\}$ in the Bochner space $L^p(\Omega_0, X)$. For $p = 2$ we simply denote $\mathrm{Rad}(X)$.

R-boundedness

- Denote the probability space $\Omega_0 = \{\pm 1\}^{\mathbb{Z}}$. Denote the k-th **Rademacher** function to be $\epsilon_k(\omega) = \omega_k$.
- For $1 \leq p < \infty$ we denote the Banach space $\mathrm{Rad}_p(X) \subseteq L^p(\Omega_0, X)$ to be the closure of the set span$\{\epsilon_k \otimes x_k : k \in \mathbb{Z}, x_k \in X\}$ in the Bochner space $L^p(\Omega_0, X)$. For $p = 2$ we simply denote $\mathrm{Rad}(X)$.
- We say $E \subseteq B(X)$ is R-bounded provided

R-boundedness

- Denote the probability space $\Omega_0 = \{\pm 1\}^{\mathbb{Z}}$. Denote the k-th **Rademacher** function to be $\epsilon_k(\omega) = \omega_k$.
- For $1 \leq p < \infty$ we denote the Banach space $\text{Rad}_p(X) \subseteq L^p(\Omega_0, X)$ to be the closure of the set $\text{span}\{\epsilon_k \otimes x_k : k \in \mathbb{Z}, x_k \in X\}$ in the Bochner space $L^p(\Omega_0, X)$. For $p = 2$ we simply denote $\text{Rad}(X)$.
- We say $E \subseteq B(X)$ is R-bounded provided

$\exists\ C > 0 \ni \forall$ finite sequence $(T_k)_{k=0}^N$ of E and $(x_k)_{k=0}^N$ of X,

$$\Big\| \sum_{k=0}^{N} \epsilon_k \otimes T_k(x_k) \Big\|_{\text{Rad(X)}} \leq C \Big\| \sum_{k=0}^{N} \epsilon_k \otimes x_k \Big\|_{\text{Rad(X)}}. \tag{1}$$

6 R-boundedness

- Denote the probability space $\Omega_0 = \{\pm 1\}^{\mathbb{Z}}$. Denote the k-th **Rademacher** function to be $\epsilon_k(\omega) = \omega_k$.
- For $1 \leq p < \infty$ we denote the Banach space $\text{Rad}_p(X) \subseteq L^p(\Omega_0, X)$ to be the closure of the set $\text{span}\{\epsilon_k \otimes x_k : k \in \mathbb{Z}, x_k \in X\}$ in the Bochner space $L^p(\Omega_0, X)$. For $p = 2$ we simply denote $\text{Rad}(X)$.
- We say $E \subseteq B(X)$ is R-bounded provided

$\exists\, C > 0 \ni \forall$ finite sequence $(T_k)_{k=0}^N$ of E and $(x_k)_{k=0}^N$ of X,

$$\Big\| \sum_{k=0}^{N} \epsilon_k \otimes T_k(x_k) \Big\|_{\text{Rad(X)}} \leq C \Big\| \sum_{k=0}^{N} \epsilon_k \otimes x_k \Big\|_{\text{Rad(X)}}. \tag{1}$$

- For the notion of R-Ritt we need to replace 'boundedness' by 'R-boundedness' in the definition of Ritt operators.

7 A big dilation theorem

Theorem (Mohanty–Ray, 2017). *Let $1 < p < \infty$. Let X be a reflexive Banach space such that both X and X^* have finite cotype.*

A big dilation theorem

Theorem (Mohanty–Ray, 2017). *Let $1 < p < \infty$. Let X be a reflexive Banach space such that both X and X^* have finite cotype. Let $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of Ritt operators on X which admits a joint bounded H^∞-functional calculus.*

A big dilation theorem

Theorem (Mohanty–Ray, 2017). *Let $1 < p < \infty$. Let X be a reflexive Banach space such that both X and X^* have finite cotype. Let $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of Ritt operators on X which admits a joint bounded H^∞-functional calculus. Then, there exists a measure space Ω, a commuting tuple of isometric isomorphisms $\mathbf{U} = (U_1, \ldots, U_n)$ on $L^p(\Omega, X)$, together with two bounded operators $\mathcal{Q} : L^p(\Omega, X) \to X$ and $\mathcal{J} : X \to L^p(\Omega, X)$ such that*

$$T_1^{i_1} \cdots T_n^{i_n} = \mathcal{Q} U_1^{i_1} \cdots U_n^{i_n} \mathcal{J} \text{ for all } i_1, \ldots, i_n \in \mathbb{N}_0.$$

A big dilation theorem

Theorem (Mohanty–Ray, 2017). *Let $1 < p < \infty$. Let X be a reflexive Banach space such that both X and X^* have finite cotype. Let $\mathbf{T} = (T_1, \dots, T_n)$ be a commuting tuple of Ritt operators on X which admits a joint bounded H^∞-functional calculus. Then, there exists a measure space Ω, a commuting tuple of isometric isomorphisms $\mathbf{U} = (U_1, \dots, U_n)$ on $L^p(\Omega, X)$, together with two bounded operators $\mathcal{Q} : L^p(\Omega, X) \to X$ and $\mathcal{J} : X \to L^p(\Omega, X)$ such that*

$$T_1^{i_1} \cdots T_n^{i_n} = \mathcal{Q} U_1^{i_1} \cdots U_n^{i_n} \mathcal{J} \text{ for all } i_1, \dots, i_n \in \mathbb{N}_0.$$

A big dilation theorem

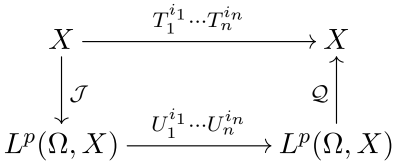
Theorem (Mohanty–Ray, 2017). *Let $1 < p < \infty$. Let X be a reflexive Banach space such that both X and X^* have finite cotype. Let $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of Ritt operators on X which admits a joint bounded H^∞-functional calculus. Then, there exists a measure space Ω, a commuting tuple of isometric isomorphisms $\mathbf{U} = (U_1, \ldots, U_n)$ on $L^p(\Omega, X)$, together with two bounded operators $\mathcal{Q} : L^p(\Omega, X) \to X$ and $\mathcal{J} : X \to L^p(\Omega, X)$ such that*

$$T_1^{i_1} \cdots T_n^{i_n} = \mathcal{Q} U_1^{i_1} \cdots U_n^{i_n} \mathcal{J} \text{ for all } i_1, \ldots, i_n \in \mathbb{N}_0.$$

A big dilation theorem

Theorem (Mohanty–Ray, 2017). *Let $1 < p < \infty$. Let X be a reflexive Banach space such that both X and X^* have finite cotype. Let $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of Ritt operators on X which admits a joint bounded H^∞-functional calculus. Then, there exists a measure space Ω, a commuting tuple of isometric isomorphisms $\mathbf{U} = (U_1, \ldots, U_n)$ on $L^p(\Omega, X)$, together with two bounded operators $\mathcal{Q} : L^p(\Omega, X) \to X$ and $\mathcal{J} : X \to L^p(\Omega, X)$ such that*

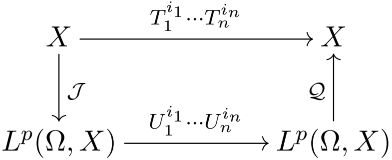
$$T_1^{i_1} \cdots T_n^{i_n} = \mathcal{Q} U_1^{i_1} \cdots U_n^{i_n} \mathcal{J} \text{ for all } i_1, \ldots, i_n \in \mathbb{N}_0.$$



A big dilation theorem

Theorem (Mohanty–Ray, 2017). *Let $1 < p < \infty$. Let X be a reflexive Banach space such that both X and X^* have finite cotype. Let $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of Ritt operators on X which admits a joint bounded H^∞-functional calculus. Then, there exists a measure space Ω, a commuting tuple of isometric isomorphisms $\mathbf{U} = (U_1, \ldots, U_n)$ on $L^p(\Omega, X)$, together with two bounded operators $\mathcal{Q} : L^p(\Omega, X) \to X$ and $\mathcal{J} : X \to L^p(\Omega, X)$ such that*

$$T_1^{i_1} \cdots T_n^{i_n} = \mathcal{Q} U_1^{i_1} \cdots U_n^{i_n} \mathcal{J} \text{ for all } i_1, \ldots, i_n \in \mathbb{N}_0.$$



- The above result is a multivariate analogoue of a similar theorem proved by Arhancet, Fackler and **Le Merdy.**

Sectorial operators

Sectorial operator: For $\omega \in (0, \pi)$, let $\Sigma_\omega := \{z \in \mathbb{C} \setminus \{0\} : |argz| < \omega\}$ be the open sector of an angle 2ω around the positive real axis $(0, \infty)$.

Sectorial operators

Sectorial operator: For $\omega \in (0,\pi)$, let $\Sigma_\omega := \{z \in \mathbb{C} \setminus \{0\} : |argz| < \omega\}$ be the open sector of an angle 2ω around the positive real axis $(0,\infty)$. A densely defined closed operator $A : D(A) \subseteq X \to X$ is sectorial of type $\omega \in (0,\pi)$

Sectorial operators

Sectorial operator: For $\omega \in (0, \pi)$, let $\Sigma_\omega := \{z \in \mathbb{C} \setminus \{0\} : |argz| < \omega\}$ be the open sector of an angle 2ω around the positive real axis $(0, \infty)$. A densely defined closed operator $A : D(A) \subseteq X \to X$ is sectorial of type $\omega \in (0, \pi)$ if, we have

(1) $\sigma(A) \subseteq \overline{\Sigma_\omega}$.

(2) For any $\nu \in (\omega, \pi)$, the set $\{zR(z, A) : z \in \mathbb{C} \setminus \overline{\Sigma_\nu}\}$ is bounded.

Sectorial operators

Sectorial operator: For $\omega \in (0, \pi)$, let $\Sigma_\omega := \{z \in \mathbb{C} \setminus \{0\} : |argz| < \omega\}$ be the open sector of an angle 2ω around the positive real axis $(0, \infty)$. A densely defined closed operator $A : D(A) \subseteq X \to X$ is sectorial of type $\omega \in (0, \pi)$ if, we have

(1) $\sigma(A) \subseteq \overline{\Sigma_\omega}$.

(2) For any $\nu \in (\omega, \pi)$, the set $\{zR(z, A) : z \in \mathbb{C} \setminus \overline{\Sigma_\nu}\}$ is bounded.

Functional calculus associated to sectorial operators:

Sectorial operators

Sectorial operator: For $\omega \in (0, \pi)$, let $\Sigma_\omega := \{z \in \mathbb{C} \setminus \{0\} : |argz| < \omega\}$ be the open sector of an angle 2ω around the positive real axis $(0, \infty)$. A densely defined closed operator $A : D(A) \subseteq X \to X$ is sectorial of type $\omega \in (0, \pi)$ if, we have

(1) $\sigma(A) \subseteq \overline{\Sigma_\omega}$.

(2) For any $\nu \in (\omega, \pi)$, the set $\{zR(z, A) : z \in \mathbb{C} \setminus \overline{\Sigma_\nu}\}$ is bounded.

Functional calculus associated to sectorial operators: Pioneered by **Mcintosh** and his coauthors.

Sectorial operators

Sectorial operator: For $\omega \in (0,\pi)$, let $\Sigma_\omega := \{z \in \mathbb{C} \setminus \{0\} : |argz| < \omega\}$ be the open sector of an angle 2ω around the positive real axis $(0,\infty)$. A densely defined closed operator $A : D(A) \subseteq X \to X$ is sectorial of type $\omega \in (0,\pi)$ if, we have

(1) $\sigma(A) \subseteq \overline{\Sigma_\omega}$.

(2) For any $\nu \in (\omega,\pi)$, the set $\{zR(z,A) : z \in \mathbb{C} \setminus \overline{\Sigma_\nu}\}$ is bounded.

Functional calculus associated to sectorial operators: Pioneered by **Mcintosh** and his coauthors.

- Let $\nu \in (0,\pi)$ and Γ_ν be the boundary of Σ_ν oriented counter-clockwise.

Sectorial operators

Sectorial operator: For $\omega \in (0,\pi)$, let $\Sigma_\omega := \{z \in \mathbb{C} \setminus \{0\} : |argz| < \omega\}$ be the open sector of an angle 2ω around the positive real axis $(0,\infty)$. A densely defined closed operator $A : D(A) \subseteq X \to X$ is sectorial of type $\omega \in (0,\pi)$ if, we have

(1) $\sigma(A) \subseteq \overline{\Sigma_\omega}$.

(2) For any $\nu \in (\omega,\pi)$, the set $\{zR(z,A) : z \in \mathbb{C} \setminus \overline{\Sigma_\nu}\}$ is bounded.

Functional calculus associated to sectorial operators: Pioneered by **Mcintosh** and his coauthors.

- Let $\nu \in (0,\pi)$ and Γ_ν be the boundary of Σ_ν oriented counter-clockwise.
- For $\theta_i \in (0,\pi)$, $1 \leq i \leq n$, denote $H_0^\infty\big(\prod_{i=1}^n \Sigma_{\theta_i}\big)$ to be the set of all bounded holomorphic functions $f : \prod_{i=1}^n \Sigma_{\theta_i} \to \mathbb{C}$.

Sectorial operators

Sectorial operator: For $\omega \in (0,\pi)$, let $\Sigma_\omega := \{z \in \mathbb{C} \setminus \{0\} : |argz| < \omega\}$ be the open sector of an angle 2ω around the positive real axis $(0,\infty)$. A densely defined closed operator $A : D(A) \subseteq X \to X$ is sectorial of type $\omega \in (0,\pi)$ if, we have

(1) $\sigma(A) \subseteq \overline{\Sigma_\omega}$.

(2) For any $\nu \in (\omega,\pi)$, the set $\{zR(z,A) : z \in \mathbb{C} \setminus \overline{\Sigma_\nu}\}$ is bounded.

Functional calculus associated to sectorial operators: Pioneered by **Mcintosh** and his coauthors.

- Let $\nu \in (0,\pi)$ and Γ_ν be the boundary of Σ_ν oriented counter-clockwise.
- For $\theta_i \in (0,\pi)$, $1 \le i \le n$, denote $H_0^\infty\big(\prod_{i=1}^n \Sigma_{\theta_i}\big)$ to be the set of all bounded holomorphic functions $f : \prod_{i=1}^n \Sigma_{\theta_i} \to \mathbb{C}$. such that

$$|f(z_1,\ldots,z_n)| \le C \prod_{i=1}^n \frac{|z_i|^s}{1+|z_i|^{2s}} \quad , \ \forall\ (z_1,\ldots,z_n) \in \prod_{i=1}^n \Sigma_{\theta_i}.$$

Sectorial operators

Sectorial operator: For $\omega \in (0, \pi)$, let $\Sigma_\omega := \{z \in \mathbb{C} \setminus \{0\} : |argz| < \omega\}$ be the open sector of an angle 2ω around the positive real axis $(0, \infty)$. A densely defined closed operator $A : D(A) \subseteq X \to X$ is sectorial of type $\omega \in (0, \pi)$ if, we have

(1) $\sigma(A) \subseteq \overline{\Sigma_\omega}$.

(2) For any $\nu \in (\omega, \pi)$, the set $\{zR(z, A) : z \in \mathbb{C} \setminus \overline{\Sigma_\nu}\}$ is bounded.

Functional calculus associated to sectorial operators: Pioneered by **Mcintosh** and his coauthors.

- Let $\nu \in (0, \pi)$ and Γ_ν be the boundary of Σ_ν oriented counter-clockwise.
- For $\theta_i \in (0, \pi)$, $1 \leq i \leq n$, denote $H_0^\infty\big(\prod_{i=1}^n \Sigma_{\theta_i}\big)$ to be the set of all bounded holomorphic functions $f : \prod_{i=1}^n \Sigma_{\theta_i} \to \mathbb{C}$. such that

$$|f(z_1, \dots, z_n)| \leq C \prod_{i=1}^n \frac{|z_i|^s}{1 + |z_i|^{2s}} \quad , \ \forall \ (z_1, \dots, z_n) \in \prod_{i=1}^n \Sigma_{\theta_i}.$$

- The notion of j.b.f.c. can be defined in a similar manner to that of Ritt operators.

9 Sketch of proof of the big dilation theorem

The most important tool is **Littlewood-Paley** square function associated to Ritt operators.

Sketch of proof of the big dilation theorem

The most important tool is **Littlewood-Paley** square function associated to Ritt operators.

$$\|x\|_{T,\alpha} := \Big\| \sum_{k=0}^{\infty} (k+1)^{\alpha-\frac{1}{2}} \epsilon_k \otimes T^k (I-T)^{\alpha} x \Big\|_{\mathrm{Rad}(X)}, \; x \in X.$$

Sketch of proof of the big dilation theorem

The most important tool is **Littlewood-Paley** square function associated to Ritt operators.

$$\|x\|_{T,\alpha} := \Big\| \sum_{k=0}^{\infty} (k+1)^{\alpha-\frac{1}{2}} \epsilon_k \otimes T^k (I-T)^\alpha x \Big\|_{\mathsf{Rad}(X)}, \; x \in X.$$

- Since T_1 admits a bounded $H^\infty(\mathcal{B}_\gamma)$, $\exists\, C > 0$ such that

$$\|x\|_{T,\frac{1}{2}} \leq C\|x\|_X, \;\; \text{for all } x \in X.$$

9 Sketch of proof of the big dilation theorem

The most important tool is **Littlewood-Paley** square function associated to Ritt operators.

$$\|x\|_{T,\alpha} := \Big\| \sum_{k=0}^{\infty} (k+1)^{\alpha-\frac{1}{2}} \epsilon_k \otimes T^k (I-T)^{\alpha} x \Big\|_{\mathsf{Rad}(X)}, \ x \in X.$$

- Since T_1 admits a bounded $H^\infty(\mathcal{B}_\gamma)$, $\exists\, C > 0$ such that
$$\|x\|_{T,\frac{1}{2}} \leq C\|x\|_X, \ \text{ for all } x \in X.$$

- Use the square function estimate to construct maps J_1, Q_1 and U to obtain $\forall n \geq 0$

$$\begin{array}{ccc} X & \xrightarrow{\quad T_1^n \quad} & X \\ \Big\downarrow{\scriptstyle J_1} & & \Big\uparrow{\scriptstyle Q_1} \\ X \oplus_p L^p(\Omega_0, X) & \xrightarrow{\quad U^n \quad} & X \oplus_p L^p(\Omega_0, X) \end{array}$$

$_{10}$Sketch of proof of the big dilation theorem continued...

The construction of these maps were done by Le Merdy-Fackler-Arhancet.

- $U = I_X \oplus (\mathsf{u}^n \otimes I_X)$ where $\mathsf{u} : L^p(\Omega_0) \to L^p(\Omega_0)$ as $\mathsf{u}(f)(\{\omega_k\}_k) = f(\{\omega_{k-1}\}_k)$ for $f \in L^p(\Omega_0)$.

Sketch of proof of the big dilation theorem continued...

The construction of these maps were done by Le Merdy-Fackler-Arhancet.

- $U = I_X \oplus (\mathsf{u}^n \otimes I_X)$ where $\mathsf{u} : L^p(\Omega_0) \to L^p(\Omega_0)$ as $\mathsf{u}(f)(\{\omega_k\}_k) = f(\{\omega_{k-1}\}_k)$ for $f \in L^p(\Omega_0)$.
- Use Mean Ergodic Theorem to decompose X and define the linear maps $J : \mathrm{Ker}(I_X - T_1) \oplus \overline{\mathrm{Ran}(I_X - T_1)} \to X \oplus_p L^p(\Omega_0, X)$ as

$$J(x_0 \oplus x_1) = x_0 \oplus \sum_{k=0}^{\infty} \epsilon_k \otimes T_1^k (I_X - T_1)^{\frac{1}{2}} x_1$$

Sketch of proof of the big dilation theorem continued...

The construction of these maps were done by Le Merdy-Fackler-Arhancet.

- $U = I_X \oplus (\mathsf{u}^n \otimes I_X)$ where $\mathsf{u} : L^p(\Omega_0) \to L^p(\Omega_0)$ as $\mathsf{u}(f)(\{\omega_k\}_k) = f(\{\omega_{k-1}\}_k)$ for $f \in L^p(\Omega_0)$.
- Use Mean Ergodic Theorem to decompose X and define the linear maps $J : \mathrm{Ker}(I_X - T_1) \oplus \overline{\mathrm{Ran}(I_X - T_1)} \to X \oplus_p L^p(\Omega_0, X)$ as

$$J(x_0 \oplus x_1) = x_0 \oplus \sum_{k=0}^{\infty} \epsilon_k \otimes T_1^k (I_X - T_1)^{\frac{1}{2}} x_1$$

and $\tilde{J} : \mathrm{Ker}(I_{X^*} - T_1^*) \oplus \overline{\mathrm{Ran}(I_{X^*} - T_1^*)} \to X^* \oplus_{p'} L^{p'}(\Omega_0, X^*)$ as

$$\tilde{J}_1(y_0 \oplus y_1) = y_0 \oplus \sum_{k=0}^{\infty} \epsilon_k \otimes {T_1^*}^k (I_{X^*} - T_1^*)^{\frac{1}{2}} y_1.$$

Sketch of proof of the big dilation theorem continued...

The construction of these maps were done by Le Merdy-Fackler-Arhancet.

- $U = I_X \oplus (\mathsf{u}^n \otimes I_X)$ where $\mathsf{u} : L^p(\Omega_0) \to L^p(\Omega_0)$ as $\mathsf{u}(f)(\{\omega_k\}_k) = f(\{\omega_{k-1}\}_k)$ for $f \in L^p(\Omega_0)$.
- Use Mean Ergodic Theorem to decompose X and define the linear maps $J : \mathrm{Ker}(I_X - T_1) \oplus \overline{\mathrm{Ran}(I_X - T_1)} \to X \oplus_p L^p(\Omega_0, X)$ as

$$J(x_0 \oplus x_1) = x_0 \oplus \sum_{k=0}^{\infty} \epsilon_k \otimes T_1^k (I_X - T_1)^{\frac{1}{2}} x_1$$

and $\tilde{J} : \mathrm{Ker}(I_{X^*} - T_1^*) \oplus \overline{\mathrm{Ran}(I_{X^*} - T_1^*)} \to X^* \oplus_{p'} L^{p'}(\Omega_0, X^*)$ as

$$\tilde{J}_1(y_0 \oplus y_1) = y_0 \oplus \sum_{k=0}^{\infty} \epsilon_k \otimes {T_1^*}^k (I_{X^*} - T_1^*)^{\frac{1}{2}} y_1.$$

- Define $Q_1 = \tilde{J}^*$ and $J_1 = J\Theta$ to obtain $T_1^n = Q_1 U^n J_1, n \geq 0$. $\Theta(x_0 \oplus x_1) := x_0 \oplus (I_X + T_1)x_1$.

$_{11}$Completing the proof

- From earlier constructions establish the identity

$$J_1 S = (S \oplus (I_{L^p(\Omega_0)} \otimes S)) J_1$$

where $S : X \to X$ is a bounded operator which commutes with T_1.

Completing the proof

- From earlier constructions establish the identity

$$J_1 S = (S \oplus (I_{L^p(\Omega_0)} \otimes S))J_1$$

where $S : X \to X$ is a bounded operator which commutes with T_1.

- Assume $(T_2, \ldots, T_m)$ already satisfies the big dilation theorem with a measure space Ω''. Notice that

$$T_1^{i_1} T_2^{i_2} \ldots T_m^{i_m} = \mathcal{Q}V\Big(\prod_{j=2}^{m} (U_j^{i_j} \oplus (I_{L^p(\Omega_0)} \otimes U_j^{i_j}))\Big)\mathcal{J}.$$

11 Completing the proof

- From earlier constructions establish the identity

$$J_1 S = (S \oplus (I_{L^p(\Omega_0)} \otimes S))J_1$$

where $S : X \to X$ is a bounded operator which commutes with T_1.

- Assume $(T_2, \ldots, T_m)$ already satisfies the big dilation theorem with a measure space Ω''. Notice that

$$T_1^{i_1} T_2^{i_2} \ldots T_m^{i_m} = \mathcal{Q} V \Big(\prod_{j=2}^{m} (U_j^{i_j} \oplus (I_{L^p(\Omega_0)} \otimes U_j^{i_j})) \Big) \mathcal{J}.$$

where $\mathcal{Q} = Q_1(Q_2 \oplus (I_{L^p(\Omega_0)} \otimes Q_2))$,

$$\mathcal{J} = (J_2 \oplus (I_{L^p(\Omega_0) \otimes J_2}))J_1$$

and

$$V = (I_{L^p(\Omega'', X)} \oplus (\mathsf{u}^{i_1} \otimes I_{L^p(\Omega'', X)}))$$

${}_{11}$Completing the proof

- From earlier constructions establish the identity

$$J_1 S = (S \oplus (I_{L^p(\Omega_0)} \otimes S))J_1$$

where $S : X \to X$ is a bounded operator which commutes with T_1.

- Assume $(T_2, \ldots, T_m)$ already satisfies the big dilation theorem with a measure space Ω''. Notice that

$$T_1^{i_1} T_2^{i_2} \ldots T_m^{i_m} = \mathcal{Q} V \Big(\prod_{j=2}^{m} (U_j^{i_j} \oplus (I_{L^p(\Omega_0)} \otimes U_j^{i_j})) \Big) \mathcal{J}.$$

where $\mathcal{Q} = Q_1(Q_2 \oplus (I_{L^p(\Omega_0)} \otimes Q_2))$,

$$\mathcal{J} = (J_2 \oplus (I_{L^p(\Omega_0) \otimes J_2}))J_1$$

and

$$V = (I_{L^p(\Omega'', X)} \oplus (\mathsf{u}^{i_1} \otimes I_{L^p(\Omega'', X)}))$$

Proof is completed by induction.

Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle.

Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle. Let G be a locally compact abelian group. Let $R : G \to B(L^p(\Omega, \mathbb{F}, \mu))$ satisfies the following conditions:

Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle. Let G be a locally compact abelian group. Let $R : G \to B(L^p(\Omega, \mathbb{F}, \mu))$ satisfies the following conditions:
  - (1) For each $f \in L^p(\Omega, \mathbb{F}, \mu)$, the map $u \mapsto R_u f$ is continuous.

Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle. Let G be a locally compact abelian group. Let $R : G \to B(L^p(\Omega, \mathbb{F}, \mu))$ satisfies the following conditions:
  (1) For each $f \in L^p(\Omega, \mathbb{F}, \mu)$, the map $u \mapsto R_u f$ is continuous.
  (2) The quantity $C_R := \sup_u \|R_u\|_{L^p(\Omega,\mathbb{F},\mu) \to L^p(\Omega,\mathbb{F},\mu)}$ is finite.

Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle. Let G be a locally compact abelian group. Let $R : G \to B(L^p(\Omega, \mathbb{F}, \mu))$ satisfies the following conditions:
  (1) For each $f \in L^p(\Omega, \mathbb{F}, \mu)$, the map $u \mapsto R_u f$ is continuous.
  (2) The quantity $C_R := \sup_u \|R_u\|_{L^p(\Omega,\mathbb{F},\mu) \to L^p(\Omega,\mathbb{F},\mu)}$ is finite.
  (3) For all $u, v \in G$, $R_u R_v = R_{uv}$.

Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle. Let G be a locally compact abelian group. Let $R : G \to B(L^p(\Omega, \mathbb{F}, \mu))$ satisfies the following conditions:
  1. For each $f \in L^p(\Omega, \mathbb{F}, \mu)$, the map $u \mapsto R_u f$ is continuous.
  2. The quantity $C_R := \sup_u \|R_u\|_{L^p(\Omega,\mathbb{F},\mu) \to L^p(\Omega,\mathbb{F},\mu)}$ is finite.
  3. For all $u, v \in G$, $R_u R_v = R_{uv}$.

Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle. Let G be a locally compact abelian group. Let $R : G \to B(L^p(\Omega, \mathbb{F}, \mu))$ satisfies the following conditions:
  1. For each $f \in L^p(\Omega, \mathbb{F}, \mu)$, the map $u \mapsto R_u f$ is continuous.
  2. The quantity $C_R := \sup_u \|R_u\|_{L^p(\Omega,\mathbb{F},\mu)\to L^p(\Omega,\mathbb{F},\mu)}$ is finite.
  3. For all $u, v \in G$, $R_u R_v = R_{uv}$.

For all $k \in L^1(G)$ with compact support define
$H_k f := \int_G k(u) R_u f, f \in L^p(\Omega, \mathbb{F}, \mu)$.Then $\|H_k\|_{L^p(\Omega,\mathbb{F},\mu)\to L^p(\Omega,\mathbb{F},\mu)} \leq C_R^2 N_p(k)$

Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle. Let G be a locally compact abelian group. Let $R : G \to B(L^p(\Omega, \mathbb{F}, \mu))$ satisfies the following conditions:
  (1) For each $f \in L^p(\Omega, \mathbb{F}, \mu)$, the map $u \mapsto R_u f$ is continuous.
  (2) The quantity $C_R := \sup_u \|R_u\|_{L^p(\Omega,\mathbb{F},\mu)\to L^p(\Omega,\mathbb{F},\mu)}$ is finite.
  (3) For all $u, v \in G$, $R_u R_v = R_{uv}$.

For all $k \in L^1(G)$ with compact support define $H_k f := \int_G k(u) R_u f, f \in L^p(\Omega, \mathbb{F}, \mu)$.Then $\|H_k\|_{L^p(\Omega,\mathbb{F},\mu)\to L^p(\Omega,\mathbb{F},\mu)} \leq C_R^2 N_p(k)$ where $N_p(k)$ is the operator norm of the convolution operator $f \mapsto k * f$ on $L^p(G)$.

12 Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle. Let G be a locally compact abelian group. Let $R : G \to B(L^p(\Omega, \mathbb{F}, \mu))$ satisfies the following conditions:
  1. For each $f \in L^p(\Omega, \mathbb{F}, \mu)$, the map $u \mapsto R_u f$ is continuous.
  2. The quantity $C_R := \sup_u \|R_u\|_{L^p(\Omega,\mathbb{F},\mu)\to L^p(\Omega,\mathbb{F},\mu)}$ is finite.
  3. For all $u, v \in G$, $R_u R_v = R_{uv}$.

For all $k \in L^1(G)$ with compact support define $H_k f := \int_G k(u) R_u f$, $f \in L^p(\Omega, \mathbb{F}, \mu)$.Then $\|H_k\|_{L^p(\Omega,\mathbb{F},\mu)\to L^p(\Omega,\mathbb{F},\mu)} \leq C_R^2 N_p(k)$ where $N_p(k)$ is the operator norm of the convolution operator $f \mapsto k * f$ on $L^p(G)$.

From Transference to von Neumann inequality: Let $\mathbf{T} = (T_1, \dots, T_n)$ be commuting tuple of bounded operators on L^p-space, $1 < p < \infty$ which admits a joint isometric loose dilation

Transfer Principles

We use two transfer principles.

- Coifman-Weiss Transference Principle. Let G be a locally compact abelian group. Let $R : G \to B(L^p(\Omega, \mathbb{F}, \mu))$ satisfies the following conditions:
  (1) For each $f \in L^p(\Omega, \mathbb{F}, \mu)$, the map $u \mapsto R_u f$ is continuous.
  (2) The quantity $C_R := \sup_u \|R_u\|_{L^p(\Omega,\mathbb{F},\mu)\to L^p(\Omega,\mathbb{F},\mu)}$ is finite.
  (3) For all $u, v \in G$, $R_u R_v = R_{uv}$.

For all $k \in L^1(G)$ with compact support define $H_k f := \int_G k(u) R_u f, f \in L^p(\Omega, \mathbb{F}, \mu)$.Then $\|H_k\|_{L^p(\Omega,\mathbb{F},\mu)\to L^p(\Omega,\mathbb{F},\mu)} \leq C_R^2 N_p(k)$ where $N_p(k)$ is the operator norm of the convolution operator $f \mapsto k * f$ on $L^p(G)$.

From Transference to von Neumann inequality: Let $\mathbf{T} = (T_1, \ldots, T_n)$ be commuting tuple of bounded operators on L^p-space, $1 < p < \infty$ which admits a joint isometric loose dilation then $\mathbf{T}$ is jontly p-polynomially bounded i.e., $\forall\ P \in \mathbb{C}[Z_1, \ldots, Z_n]$

$$\|P(\mathbf{T})\|_{L^p \to L^p} \leq C \|P(S_1, \ldots, S_n)\|_{\ell_p(\mathbb{Z}^n)\to\ell_p(\mathbb{Z}^n)}$$

.

Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017).

13 Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017). *Suppose* $\mathbf{T} = (T_1, \ldots, T_n)$ *is a commuting tuple of Ritt operators on* X. *Let us denote the sectorial operators* $A_i = I_X - T_i$ *for* $1 \leq i \leq n$.

Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017). *Suppose* $\mathbf{T} = (T_1, \ldots, T_n)$ *is a commuting tuple of Ritt operators on* X*. Let us denote the sectorial operators* $A_i = I_X - T_i$ *for* $1 \leq i \leq n$*. The tuple* $\mathbf{T}$ *admits a joint bounded* $H^\infty(\prod_{i=1}^{n} \mathcal{B}_{\gamma_i})$ *functional calculus for some* $\gamma_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$ *if and only if*

Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017). *Suppose $\mathbf{T} = (T_1, \ldots, T_n)$ is a commuting tuple of Ritt operators on X. Let us denote the sectorial operators $A_i = I_X - T_i$ for $1 \leq i \leq n$. The tuple $\mathbf{T}$ admits a joint bounded $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i})$ functional calculus for some $\gamma_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$ if and only if the tuple $\mathbf{A} = (A_1, \ldots, A_n)$ admits a joint bounded $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ functional calculus for some $\theta_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$.*

Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017). *Suppose* $\mathbf{T} = (T_1, \ldots, T_n)$ *is a commuting tuple of Ritt operators on* X*. Let us denote the sectorial operators* $A_i = I_X - T_i$ *for* $1 \leq i \leq n$*. The tuple* $\mathbf{T}$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i})$ *functional calculus for some* $\gamma_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$ *if and only if the tuple* $\mathbf{A} = (A_1, \ldots, A_n)$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ *functional calculus for some* $\theta_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$.

- The above result generalizes Le Merdy's one variable transfer principle.

Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017). *Suppose* $\mathbf{T} = (T_1, \ldots, T_n)$ *is a commuting tuple of Ritt operators on* X. *Let us denote the sectorial operators* $A_i = I_X - T_i$ *for* $1 \leq i \leq n$. *The tuple* $\mathbf{T}$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i})$ *functional calculus for some* $\gamma_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$ *if and only if the tuple* $\mathbf{A} = (A_1, \ldots, A_n)$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ *functional calculus for some* $\theta_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$.

- The above result generalizes Le Merdy's one variable transfer principle.
- A crucial fact we will be using that L^p-spaces have the $\mathfrak{n}$-f.c.p.

13 Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017). *Suppose* $\mathbf{T} = (T_1, \ldots, T_n)$ *is a commuting tuple of Ritt operators on* X. *Let us denote the sectorial operators* $A_i = I_X - T_i$ *for* $1 \leq i \leq n$. *The tuple* $\mathbf{T}$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i})$ *functional calculus for some* $\gamma_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$ *if and only if the tuple* $\mathbf{A} = (A_1, \ldots, A_n)$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ *functional calculus for some* $\theta_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$.

- The above result generalizes Le Merdy's one variable transfer principle.
- A crucial fact we will be using that L^p-spaces have the $\mathfrak{n}$-f.c.p.

13 Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017). *Suppose* $\mathbf{T} = (T_1, \ldots, T_n)$ *is a commuting tuple of Ritt operators on* X. *Let us denote the sectorial operators* $A_i = I_X - T_i$ *for* $1 \leq i \leq n$. *The tuple* $\mathbf{T}$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i})$ *functional calculus for some* $\gamma_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$ *if and only if the tuple* $\mathbf{A} = (A_1, \ldots, A_n)$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ *functional calculus for some* $\theta_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$.

- The above result generalizes Le Merdy's one variable transfer principle.
- A crucial fact we will be using that L^p-spaces have the $\mathfrak{n}$-f.c.p.

Theorem (Mohanty–Ray, 2017).

Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017). *Suppose* $\mathbf{T} = (T_1, \ldots, T_n)$ *is a commuting tuple of Ritt operators on* X*. Let us denote the sectorial operators* $A_i = I_X - T_i$ *for* $1 \leq i \leq n$*. The tuple* $\mathbf{T}$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i})$ *functional calculus for some* $\gamma_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$ *if and only if the tuple* $\mathbf{A} = (A_1, \ldots, A_n)$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ *functional calculus for some* $\theta_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$.

- The above result generalizes Le Merdy's one variable transfer principle.
- A crucial fact we will be using that L^p-spaces have the $\mathfrak{n}$-f.c.p.

Theorem (Mohanty–Ray, 2017). *Let* $\mathbf{T} = (T_1, \ldots, T_n)$ *be a commuting tuple of positive contractions on* L^p*-spaces* ($1 < p < \infty$) *which are Ritt operators. Then* $\mathbf{T}$ *admits a joint isometric loose dilation.*

Theorem on transferring the bounded j.f.c.

Theorem (Mohanty–Ray, 2017). *Suppose* $\mathbf{T} = (T_1, \ldots, T_n)$ *is a commuting tuple of Ritt operators on* X*. Let us denote the sectorial operators* $A_i = I_X - T_i$ *for* $1 \leq i \leq n$*. The tuple* $\mathbf{T}$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i})$ *functional calculus for some* $\gamma_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$ *if and only if the tuple* $\mathbf{A} = (A_1, \ldots, A_n)$ *admits a joint bounded* $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ *functional calculus for some* $\theta_i \in (0, \frac{\pi}{2})$, $1 \leq i \leq n$.

- The above result generalizes Le Merdy's one variable transfer principle.
- A crucial fact we will be using that L^p-spaces have the $\mathfrak{n}$-f.c.p.

Theorem (Mohanty–Ray, 2017). *Let* $\mathbf{T} = (T_1, \ldots, T_n)$ *be a commuting tuple of positive contractions on* L^p*-spaces* ($1 < p < \infty$) *which are Ritt operators. Then* $\mathbf{T}$ *admits a joint isometric loose dilation.*

- The above result can be thought of a weak analogoue of multivariate Akcoglu's dilation theorem.

A characterization of joint bounded f.c.

Theorem (Mohanty–Ray, 2017). *Let* $1 < p \neq 2 < \infty$ *and* $\mathbf{T} = (T_1, \ldots, T_n)$ *be a commuting tuple of Ritt operators on* $L^p(\Omega)$*. Then the following assertions are equivalent.*

A characterization of joint bounded f.c.

Theorem (Mohanty–Ray, 2017). *Let $1 < p \neq 2 < \infty$ and $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of Ritt operators on $L^p(\Omega)$. Then the following assertions are equivalent.*

1. *The tuple $\mathbf{T}$ admits a joint bounded $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i})$, $\gamma_i \in (0, \frac{\pi}{2})$, functional calculus.*

A characterization of joint bounded f.c.

Theorem (Mohanty–Ray, 2017). *Let $1 < p \neq 2 < \infty$ and $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of Ritt operators on $L^p(\Omega)$. Then the following assertions are equivalent.*

1. *The tuple $\mathbf{T}$ admits a joint bounded $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i}),\ \gamma_i \in (0, \frac{\pi}{2})$, functional calculus.*
2. *Each T_i, $1 \leq i \leq n$ is R-Ritt and $\mathbf{T}$ admits a joint isometric loose dilation.*

A characterization of joint bounded f.c.

Theorem (Mohanty–Ray, 2017). *Let $1 < p \neq 2 < \infty$ and $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of Ritt operators on $L^p(\Omega)$. Then the following assertions are equivalent.*

1. *The tuple $\mathbf{T}$ admits a joint bounded $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i}),\ \gamma_i \in (0, \frac{\pi}{2})$, functional calculus.*
2. *Each $T_i,\ 1 \leq i \leq n$ is R-Ritt and $\mathbf{T}$ admits a joint isometric loose dilation.*
3. *Each $T_i,\ 1 \leq i \leq n$ is R-Ritt and $\mathbf{T}$ is jointly p-polynomially bounded.*

A characterization of joint bounded f.c.

Theorem (Mohanty–Ray, 2017). *Let $1 < p \neq 2 < \infty$ and $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of Ritt operators on $L^p(\Omega)$. Then the following assertions are equivalent.*

1. *The tuple $\mathbf{T}$ admits a joint bounded $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i}),\ \gamma_i \in (0, \frac{\pi}{2})$, functional calculus.*
2. *Each T_i, $1 \leq i \leq n$ is R-Ritt and $\mathbf{T}$ admits a joint isometric loose dilation.*
3. *Each T_i, $1 \leq i \leq n$ is R-Ritt and $\mathbf{T}$ is jointly p-polynomially bounded.*
4. *Each T_i, $1 \leq i \leq n$ is R-Ritt and $(I - T_1, \ldots, I - T_n)$ admits a joint bounded $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ functional calculus for $\theta_i \in (0, \pi)$ for each $1 \leq i \leq n$.*

A characterization of joint bounded f.c.

Theorem (Mohanty–Ray, 2017). *Let $1 < p \neq 2 < \infty$ and $\mathbf{T} = (T_1, \dots, T_n)$ be a commuting tuple of Ritt operators on $L^p(\Omega)$. Then the following assertions are equivalent.*

1. *The tuple $\mathbf{T}$ admits a joint bounded $H^\infty(\prod_{i=1}^n \mathcal{B}_{\gamma_i}),\ \gamma_i \in (0, \frac{\pi}{2})$, functional calculus.*
2. *Each T_i, $1 \leq i \leq n$ is R-Ritt and $\mathbf{T}$ admits a joint isometric loose dilation.*
3. *Each T_i, $1 \leq i \leq n$ is R-Ritt and $\mathbf{T}$ is jointly p-polynomially bounded.*
4. *Each T_i, $1 \leq i \leq n$ is R-Ritt and $(I - T_1, \dots, I - T_n)$ admits a joint bounded $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ functional calculus for $\theta_i \in (0, \pi)$ for each $1 \leq i \leq n$.*

- The above result generalizes single variable characterizations of Le Merdy-arhancet and Le Merdy-Fackler-Arhancet.

Joint similarity problem

(Pisier,1997) There exists a polynomially bounded operator on a Hilbert space which is not similar to a contraction.**(This resolves a long-standing open problem of Paul Halmos.)**

Joint similarity problem

(Pisier,1997) There exists a polynomially bounded operator on a Hilbert space which is not similar to a contraction.**(This resolves a long-standing open problem of Paul Halmos.)**

Joint similarity problem

(Pisier,1997) There exists a polynomially bounded operator on a Hilbert space which is not similar to a contraction.**(This resolves a long-standing open problem of Paul Halmos.)**

(Pisier,1998) There exists two commuting bounded operators on a Hilbert space, each of which being similar to a contractions but they are not jointly similar to contractions.

Joint similarity problem

(Pisier,1997) There exists a polynomially bounded operator on a Hilbert space which is not similar to a contraction.**(This resolves a long-standing open problem of Paul Halmos.)**

(Pisier,1998) **There exists two commuting bounded operators on a Hilbert space, each of which being similar to a contractions but they are not jointly similar to contractions.**

- Le Merdy showed Ritt operators are stable under Halmos similaity problem.

Joint similarity problem

(Pisier,1997) There exists a polynomially bounded operator on a Hilbert space which is not similar to a contraction.**(This resolves a long-standing open problem of Paul Halmos.)**

(Pisier,1998) **There exists two commuting bounded operators on a Hilbert space, each of which being similar to a contractions but they are not jointly similar to contractions.**

- Le Merdy showed Ritt operators are stable under Halmos similaity problem.

Theorem(Mohanty-Ray, 2017): Let $\mathbf{T} = (T_1, \ldots, T_n)$ commuting Ritt operators on $\mathcal{H}$. TFAE

1. Each T_i is similar to a contraction, $1 \leq i \leq n$.

Joint similarity problem

(Pisier,1997) There exists a polynomially bounded operator on a Hilbert space which is not similar to a contraction.**(This resolves a long-standing open problem of Paul Halmos.)**

(Pisier,1998) There exists two commuting bounded operators on a Hilbert space, each of which being similar to a contractions but they are not jointly similar to contractions.

- Le Merdy showed Ritt operators are stable under Halmos similaity problem.

Theorem(Mohanty-Ray, 2017): Let $\mathbf{T} = (T_1, \ldots, T_n)$ commuting Ritt operators on $\mathcal{H}$. TFAE

1. Each T_i is similar to a contraction, $1 \leq i \leq n$.
2. The tuple $\mathbf{T} = (T_1, \ldots, T_n)$ admits a j.b.f.c.

Joint similarity problem

(Pisier,1997) There exists a polynomially bounded operator on a Hilbert space which is not similar to a contraction.**(This resolves a long-standing open problem of Paul Halmos.)**

(Pisier,1998) There exists two commuting bounded operators on a Hilbert space, each of which being similar to a contractions but they are not jointly similar to contractions.

- Le Merdy showed Ritt operators are stable under Halmos similaity problem.

Theorem(Mohanty-Ray, 2017): Let $\mathbf{T} = (T_1, \ldots, T_n)$ commuting Ritt operators on $\mathcal{H}$. TFAE

1. Each T_i is similar to a contraction, $1 \leq i \leq n$.
2. The tuple $\mathbf{T} = (T_1, \ldots, T_n)$ admits a j.b.f.c.
3. The tuple $\mathbf{T} = (T_1, \ldots, T_n)$ is jointly similar to a commuting n-tuple of contractions.

On general Banach space and non-commutative L^p-space

Theorem(Mohanty-Ray, 2017). *Let $\mathbf{T} = (T_1, \cdots, T_n)$ be a commuting tuple of bounded operators on X, which is jointly p-polynomially bounded.*

On general Banach space and non-commutative L^p-space

Theorem(Mohanty-Ray, 2017). *Let* $\mathbf{T} = (T_1, \cdots, T_n)$ *be a commuting tuple of bounded operators on* X*, which is jointly* p*-polynomially bounded. Then* $(I - T_1, \ldots, I - T_n)$ *is a commuting tuple of sectorial operators and admits a joint bounded* $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ *functional calculus for all* $\theta_i \in (\frac{\pi}{2}, \pi)$*.*

On general Banach space and non-commutative L^p-space

Theorem(Mohanty-Ray, 2017). *Let $\mathbf{T} = (T_1, \cdots, T_n)$ be a commuting tuple of bounded operators on X, which is jointly p-polynomially bounded. Then $(I - T_1, \ldots, I - T_n)$ is a commuting tuple of sectorial operators and admits a joint bounded $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ functional calculus for all $\theta_i \in (\frac{\pi}{2}, \pi)$.*

- This generalizes a single variable result of Le Merdy and Arhancet. The main ingredient of the proof is Marcinkiewicz multiplier theorem.

On general Banach space and non-commutative L^p-space

Theorem(Mohanty-Ray, 2017). *Let $\mathbf{T} = (T_1, \cdots, T_n)$ be a commuting tuple of bounded operators on X, which is jointly p-polynomially bounded. Then $(I - T_1, \ldots, I - T_n)$ is a commuting tuple of sectorial operators and admits a joint bounded $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ functional calculus for all $\theta_i \in (\frac{\pi}{2}, \pi)$.*

- This generalizes a single variable result of Le Merdy and Arhancet. The main ingredient of the proof is Marcinkiewicz multiplier theorem.

Theorem(Mohanty-Ray, 2017): Let $\mathscr{M}$ be a von Neumann algebra equipped with a normal faithful semifinite trace and $1 < p \neq 2 < \infty$ and $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of R-Ritt operators on $L^p(\mathscr{M})$. Then we have $(1) \implies (2) \implies (3)$.

1. The tuple $\mathbf{T}$ admits a joint bounded functional calculus.

On general Banach space and non-commutative L^p-space

Theorem(Mohanty-Ray, 2017). *Let $\mathbf{T} = (T_1, \cdots, T_n)$ be a commuting tuple of bounded operators on X, which is jointly p-polynomially bounded. Then $(I - T_1, \ldots, I - T_n)$ is a commuting tuple of sectorial operators and admits a joint bounded $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ functional calculus for all $\theta_i \in (\frac{\pi}{2}, \pi)$.*

- This generalizes a single variable result of Le Merdy and Arhancet. The main ingredient of the proof is Marcinkiewicz multiplier theorem.

Theorem(Mohanty-Ray, 2017): Let $\mathscr{M}$ be a von Neumann algebra equipped with a normal faithful semifinite trace and $1 < p \neq 2 < \infty$ and $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of R-Ritt operators on $L^p(\mathscr{M})$. Then we have $(1) \implies (2) \implies (3)$.

1. The tuple $\mathbf{T}$ admits a joint bounded functional calculus.
2. The tuple $\mathbf{T}$ admits a joint non-commutative loose dilation.

On general Banach space and non-commutative L^p-space

Theorem(Mohanty-Ray, 2017). *Let $\mathbf{T} = (T_1, \cdots, T_n)$ be a commuting tuple of bounded operators on X, which is jointly p-polynomially bounded. Then $(I - T_1, \ldots, I - T_n)$ is a commuting tuple of sectorial operators and admits a joint bounded $H^\infty(\prod_{i=1}^n \Sigma_{\theta_i})$ functional calculus for all $\theta_i \in (\frac{\pi}{2}, \pi)$.*

- This generalizes a single variable result of Le Merdy and Arhancet. The main ingredient of the proof is Marcinkiewicz multiplier theorem.

Theorem(Mohanty-Ray, 2017): Let $\mathscr{M}$ be a von Neumann algebra equipped with a normal faithful semifinite trace and $1 < p \neq 2 < \infty$ and $\mathbf{T} = (T_1, \ldots, T_n)$ be a commuting tuple of R-Ritt operators on $L^p(\mathscr{M})$. Then we have $(1) \implies (2) \implies (3)$.

1. The tuple $\mathbf{T}$ admits a joint bounded functional calculus.
2. The tuple $\mathbf{T}$ admits a joint non-commutative loose dilation.
3. There exists a $\mathscr{N}$ be a von Neumann algebra equipped with a normal faithful semifinite trace $\mathscr{N}$ and a constant $C > 0$ such that

References

Arhancet, Cédric and Le Merdy, Christian; *Dilation of Ritt operators on L^p-spaces.* Israel Journal of Mathematics 201.1 (2014): 373-414.

References

- Arhancet, Cédric and Le Merdy, Christian; *Dilation of Ritt operators on L^p-spaces.* Israel Journal of Mathematics 201.1 (2014): 373-414.
- Arhancet Cédric, Fackler, Stephan and Le Merdy, Christian; *Isometric dilations and H^∞ calculus for bounded analytic semigroups and Ritt operators.* Transactions of the American Mathematical Society (2017).

References

- Arhancet, Cédric and Le Merdy, Christian; *Dilation of Ritt operators on L^p-spaces.* Israel Journal of Mathematics 201.1 (2014): 373-414.
- Arhancet Cédric, Fackler, Stephan and Le Merdy, Christian; *Isometric dilations and H^∞ calculus for bounded analytic semigroups and Ritt operators.* Transactions of the American Mathematical Society (2017).
- Akcoglu, M. A. and Sucheston, Louis ; *Dilations of positive contractions on Lp spaces*. Can. J. Math. Bull 20 (1977): 285-292.

References

- Arhancet, Cédric and Le Merdy, Christian; *Dilation of Ritt operators on L^p-spaces.* Israel Journal of Mathematics 201.1 (2014): 373-414.
- Arhancet Cédric, Fackler, Stephan and Le Merdy, Christian; *Isometric dilations and H^∞ calculus for bounded analytic semigroups and Ritt operators.* Transactions of the American Mathematical Society (2017).
- Akcoglu, M. A. and Sucheston, Louis ; *Dilations of positive contractions on Lp spaces*. Can. J. Math. Bull 20 (1977): 285-292.
- Ando, Tsuyoshi; *On a pair of commutative contractions*. Acta Sci. Math.(Szeged) 24.1-2 (1963): 88-90.

References

- Arhancet, Cédric and Le Merdy, Christian; *Dilation of Ritt operators on L^p-spaces.* Israel Journal of Mathematics 201.1 (2014): 373-414.
- Arhancet Cédric, Fackler, Stephan and Le Merdy, Christian; *Isometric dilations and H^∞ calculus for bounded analytic semigroups and Ritt operators.* Transactions of the American Mathematical Society (2017).
- Akcoglu, M. A. and Sucheston, Louis ; *Dilations of positive contractions on Lp spaces*. Can. J. Math. Bull 20 (1977): 285-292.
- Ando, Tsuyoshi; *On a pair of commutative contractions*. Acta Sci. Math.(Szeged) 24.1-2 (1963): 88-90.
- Halmos, Paul R; *Ten problems in Hilbert space.* Bulletin of the American Mathematical Society 76.5 (1970): 887-933.

17 # References

- Arhancet, Cédric and Le Merdy, Christian; *Dilation of Ritt operators on L^p-spaces.* Israel Journal of Mathematics 201.1 (2014): 373-414.
- Arhancet Cédric, Fackler, Stephan and Le Merdy, Christian; *Isometric dilations and H^∞ calculus for bounded analytic semigroups and Ritt operators.* Transactions of the American Mathematical Society (2017).
- Akcoglu, M. A. and Sucheston, Louis ; *Dilations of positive contractions on Lp spaces*. Can. J. Math. Bull 20 (1977): 285-292.
- Ando, Tsuyoshi; *On a pair of commutative contractions*. Acta Sci. Math.(Szeged) 24.1-2 (1963): 88-90.
- Halmos, Paul R; *Ten problems in Hilbert space.* Bulletin of the American Mathematical Society 76.5 (1970): 887-933.
- Lancien, Florence, Gilles Lancien and Le Merdy Christian; *A joint functional calculus for sectorial operators with commuting resolvents.* Proceedings of the London Mathematical Society 77.2 (1998): 387-414.

References

McIntosh, Alan; *Operators which have an H_∞ functional calculus.* Miniconference on operator theory and partial differential equations. Centre for Mathematics and its Applications, Mathematical Sciences Institute, The Australian National University, 1986.

References

- McIntosh, Alan; *Operators which have an H_∞ functional calculus.* Miniconference on operator theory and partial differential equations. Centre for Mathematics and its Applications, Mathematical Sciences Institute, The Australian National University, 1986.
- Pisier, Gilles; *A polynomially bounded operator on Hilbert space which is not similar to a contraction.* Journal of the American Mathematical Society 10.2 (1997): 351-369.

References

McIntosh, Alan; *Operators which have an H_∞ functional calculus.* Miniconference on operator theory and partial differential equations. Centre for Mathematics and its Applications, Mathematical Sciences Institute, The Australian National University, 1986.

Pisier, Gilles; *A polynomially bounded operator on Hilbert space which is not similar to a contraction.* Journal of the American Mathematical Society 10.2 (1997): 351-369.

Pisier, Gilles; *Joint similarity problems and the generation of operator algebras with bounded length.* Integral Equations and Operator Theory 31.3 (1998): 353-370.

References

- McIntosh, Alan; *Operators which have an H_∞ functional calculus.* Miniconference on operator theory and partial differential equations. Centre for Mathematics and its Applications, Mathematical Sciences Institute, The Australian National University, 1986.
- Pisier, Gilles; *A polynomially bounded operator on Hilbert space which is not similar to a contraction.* Journal of the American Mathematical Society 10.2 (1997): 351-369.
- Pisier, Gilles; *Joint similarity problems and the generation of operator algebras with bounded length.* Integral Equations and Operator Theory 31.3 (1998): 353-370.
- Varopoulos, N. Th. *On an inequality of von Neumann and an application of the metric theory of tensor products to operators theory*. Journal of Functional Analysis 16.1 (1974): 83-100.

References

- McIntosh, Alan; *Operators which have an H_∞ functional calculus.* Miniconference on operator theory and partial differential equations. Centre for Mathematics and its Applications, Mathematical Sciences Institute, The Australian National University, 1986.
- Pisier, Gilles; *A polynomially bounded operator on Hilbert space which is not similar to a contraction.* Journal of the American Mathematical Society 10.2 (1997): 351-369.
- Pisier, Gilles; *Joint similarity problems and the generation of operator algebras with bounded length.* Integral Equations and Operator Theory 31.3 (1998): 353-370.
- Varopoulos, N. Th. *On an inequality of von Neumann and an application of the metric theory of tensor products to operators theory*. Journal of Functional Analysis 16.1 (1974): 83-100.
- Neumann, Johann Von; *Eine Spektraltheorie fr allgemeine Operatoren eines unitren Raumes. Erhard Schmidt zum 75. Geburtstag in*