Redistribution Mechanisms for Assignment of Heterogeneous Objects

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Agenda

- Motivation
  - Examples
  - Quick Recap of Mechanism Design

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  - Introduction
  - State of the Art and Research Gaps
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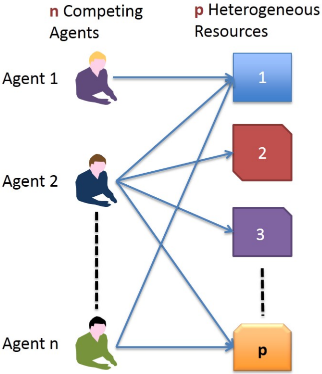
  - Problem Statement
- Contributions
- Summary and Directions for Future Work

Motivating Example

Assignment of Resources in Social Welfare Settings

Assignment of p resources among n of its users



- Telecom Regulatory Authority wishes to allot spectrum licenses
- A university wants to allot real estate to departments
- Assignment should be such that social welfare is maximized

Need for Mechanism Design

- **Critical need:** True valuations to be reported by the agents

Mechanism Design Framework is natural

- Mechanism Design is an important tool in microeconomics
- Mechanisms used in the current context are called **Redistribution Mechanisms**
  - Redistribution Mechanisms: Guo and Conitzer, "Worst Case Optimal Redistribution of VCG Payments", (ACM EC'07),
  - H Moulin, "Efficient, strategy-proof and almost budget-balanced assignment", Journal of Economic theory, 2009

Mechanism Design (MD)

Given:
a) a set of **strategic** (utility maximizing) agents with private information,
b) a social choice function that captures **desirable (social)** properties

MD provides a game theoretic framework to explore if the given social choice function may be implemented as an equilibrium outcome of an induced game.

Gibbard-Satterthwaite Impossibility Theorem

Theorem

If

1. *The outcome set* X *is such that,* $3 \leq |X| < \infty$
2. $\mathscr{R}_i = \mathscr{S} \; \forall \; i$
3. $f(\Theta) = X$, *that is, the image of SCF* $f(\cdot)$ *is the set* X.

then the social choice function SCF $f(\cdot)$ *is truthfully implementable in dominant strategies if and only if it is dictatorial.*

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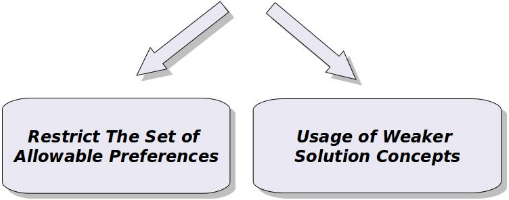
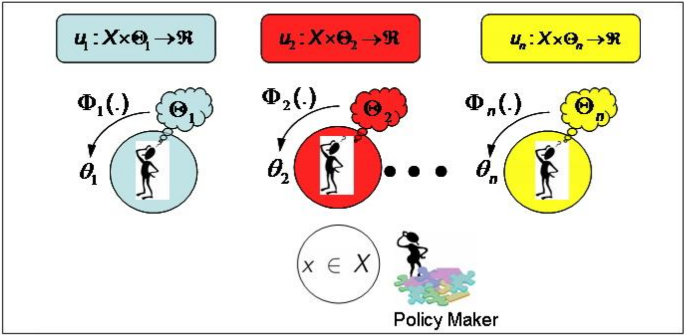


Quasi Linear Environment

$$u_i(x, \theta_i) = v_i(k, \theta_i) - t_i$$

Valuation function of agent i



$$X = \left\{ (k, t_1, \ldots, t_n) : k \in K, t_i \in \mathbb{R} \; \forall \; i = 1, \ldots, n, \sum_i t_i \leq 0 \right\}$$

Project Choice — Monetary transfer to agent i

Properties of Mechanisms

Non-Dictatorship

No single agent is favored all the time

Dominant Strategy Incentive Compatibility (DSIC)

DSIC Reporting truth is dominant strategy

AE

Allocative Efficiency Allocate item to those who value them most

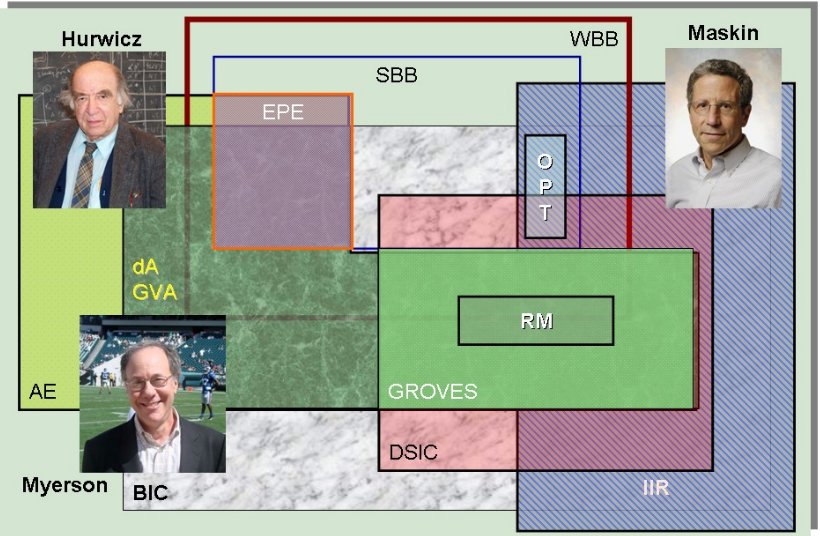
IR

Individual Rationality Agents participate voluntarily. (No losses)

SBB

Strict Budget Balance Net transfer of money is zero

Space of Mechanisms in Quasi-Linear Settings



Redistribution Mechanisms

Groves Mechanism

Recall Groves Theorem:

Theorem (Groves Theorem)

An allocatively efficient SCF f can be truthfully implemented in dominant strategies if,

$$t_i(\theta) = -\left[\sum_{j \neq i} v_j(k^*(\theta), \theta_j)\right] + h_i(\theta_{-i}); \quad i = 1, 2, \ldots, n \tag{1}$$

where $h_i(\cdot)$ is any arbitrary function of θ_{-i}.

[1] T. Groves. Incentives in teams. *Econometrica*, 41:617-631, 1973

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where $h_i(\cdot)$ is any arbitrary function of θ_{-i}.

The above payment is **Groves payment scheme.** [1]

[1] T. Groves. Incentives in teams. *Econometrica*, 41:617-631, 1973

Clarke's Mechanism

- In Groves Mechanism, let

$$h_i(\theta_{-i}) = \sum_{j \neq i} v_j(k^*_{-i}(\theta_{-i}), \theta_j); \;\; \forall\, \theta_{-i} \in \Theta_{-i}, \;\; i = 1, \ldots, n \qquad (2)$$

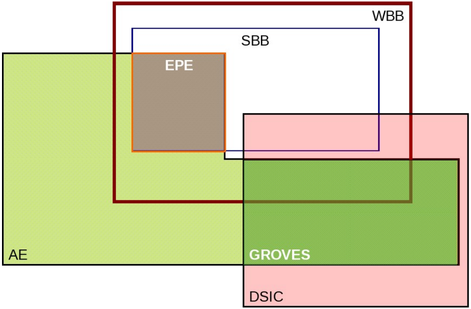
- That is, each agent i pays,

$$t_i(\theta) = -\left[\sum_{j \neq i} v_j(k^*(\theta), \theta_j)\right] + \left[\sum_{j \neq i} v_j(k^*_{-i}(\theta_{-i}), \theta_j)\right] \qquad (3)$$

- This mechanism is called **VCG** or **Clarke's[2] Pivotal Mechanism**

[2] E. Clarke. Multi-part pricing of public goods. *Public Choice*, 11:17-23, 1971

Green Laffont Impossibility Theorem



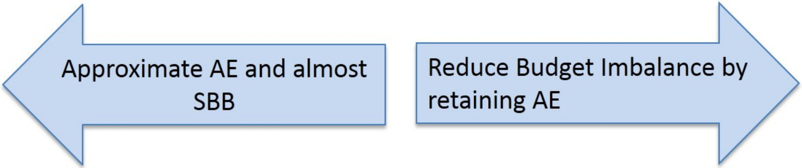
AE : Allocative Efficient
DSIC : Dominant strategy Incentive Compatible
WBB : Weak Budget Balanced
SBB: Strict Budget Balanced
EPE: Ex-post efficient

Impossibility Result

AE+SBB+DSIC is impossible to achieve

J. R. Green and J. J. Laffont, "**Incentives in Public Decision Making**". North-Holland Publishing Company, Amsterdam, 1979

Two Approaches to Design Mechanisms



- Approximate AE
  - Faltings [1]: Randomly select pool of agents who collect all the revenue
  - Guo and Conitzer [2]: Burn certain number of items
- Today's Talk: Reduce Budget Imbalance
  - Redistribution Mechanisms

Redistribution Mechanism

- Laffont and Maskin [3] [3] : redistribute the surplus

[3] J.J. Laffont and E. Maskin. A differential approach to expected utility maximizing mechanisms. In J. J Laffont, editor, Aggregation and Revelation of Preferences. 1979.

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- Redistribute the surplus that preserves AE and DSIC (Design Appropriate Groves mechanism)

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- Redistribute the surplus that preserves AE and DSIC (Design Appropriate Groves mechanism)
- Refer to it as **redistribution mechanism**

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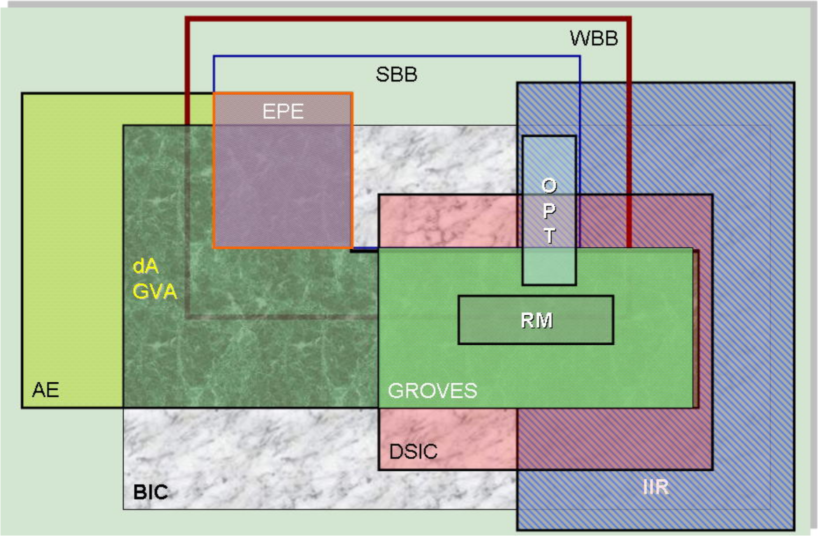
RM in Space of Mechanisms



Redistribution Mechanisms

- Groves mechanism: design payment functions $h_i(\theta_{-i})$s for all the agents.
- VCG mechanism: $h_i(\theta_{-i}) = \sum_{j \neq i} v_j(k^*_{-i}(\theta_{-i}), \theta_j)$
- Let $h_i(\theta_{-i}) = \sum_{j \neq i} v_j(k^*_{-i}(\theta_{-i}), \theta_j) - r_i(\theta_{-i})$
  where, $r_i(.)$ is rebate function for the agent i.
- A redistribution mechanism: involves designing an appropriate **rebate function**

State of the Art and Research Gaps

All the previous work assumes the objects are identical (Homogeneous settings).

- Cavallo [4][4] : rebate function that depends only on $(p+2)$ highest bids

[4] This scheme can be viewed as Bailey [5] scheme applied in the setting

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- Herve Moulin [6] : notion of efficiency loss,

$$L(n,p) = \max_{\theta \in \Theta} \frac{\text{Budget Surplus} = \sum_i t_i - r_i}{\text{Efficient Surplus} = \sum_i v_i(k^*, .)}$$

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- Guo and Conitzer [7] : performance ratio of a mechanism as,

$$\min_{\theta \in \Theta} \frac{\text{Surplus redistributed} = \sum_i r_i}{\text{VCG Surplus} = \sum_i t_i}$$

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- Guo and Conitzer [8] : mechanism which is optimal in expected sense

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Bailey-Cavello Mechanism

- Sort the received bids
- The highest p bidders get the objects
- The winners pay VCG payment: $p+1^{st}$ bid
- Rebate $r_i()$ to i^{th} agent

$$
\begin{aligned}
r_i &= \frac{1}{n} v_{p+2} \text{ if } i = 1, 2, \ldots, p+1 \\
&= \frac{1}{n} v_{p+1} \text{ if } i = p+2, p+3, \ldots, n
\end{aligned}
$$

WCO Mechanism

Moulin [6] and Guo and Conitzer [7] : Worst Case Optimal **(WCO)** Mechanism,

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$$r_i = f(\theta_1, \theta_2, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n) \tag{4}$$

where,

$$f(x_1, x_2, \ldots, x_{n-1}) = \sum_{j=p+1}^{n-1} c_j x_j$$

$$c_i = \frac{(-1)^{i+p-1}(n-p)\binom{n-1}{p-1}}{i\binom{n-1}{i}\sum_{j=p}^{n-1}\binom{n-1}{j}}\left\{\sum_{j=i}^{n-1}\binom{n-1}{j}\right\};\ i = p+1, \ldots, n-1$$

An Example of WCO Mechanism

Consider: $p = 1$ and $n = 3$.

- **VCG Mechanism**
  The highest bidder will win and pay the second highest bid
- **WCO Mechanism**
  Say, $v_1 \geq v_2 \geq v_3$

$$\boxed{\begin{array}{ccl} p_1 & = & v_2 - \frac{1}{3}v_3 \\ p_2 & = & -\frac{1}{3}v_3 \\ p_3 & = & -\frac{1}{3}v_2 \\ e & = & \frac{1}{3} \end{array}} \qquad (5)$$

Say $v_1 = 1, v_2 = 1, v_3 = 0$
$t_1 = -1, t_2 = 0, t_3 = 0, p_1 = -1, p_2 = 0 p_3 = \frac{1}{3}$
$e = \frac{\frac{1}{3}}{1} = \frac{1}{3}$

Problem We Are Addressing

- p heterogeneous objects to be assigned among n competing agents, where $n > p$ and agents have unit demand
- All the previous work assumes objects are homogeneous

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- p heterogeneous objects to be assigned among n competing agents, where $n > p$ and agents have unit demand
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Goal

Design a **redistribution mechanism** which is individually rational, feasible and worst case optimal for assignment of p heterogeneous objects among n agents with unit demand.

Problem Formulation

t	$=$	$\sum_i t_i =$ VCG Surplus
$\sum_i r_i(.)$	$=$	Surplus redistributed
m	$=$	$(r_1(.), r_2(.), \ldots, r_n(.))$ a redistribution mechanism
$e(m)$	$=$	$\min_{\theta \in \Theta, t \neq 0} \frac{\sum_i r_i(.)}{t}$ **Redistribution Index**
$\mathscr{M}$	:	The space of all anonymous redistribution mechanisms for assignment of heterogeneous objects.

Problem Formulation

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\begin{array}{lcl}
t & = & \sum_i t_i = \text{VCG Surplus} \\
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\mathscr{M} & : & \text{The space of all anonymous redistribution mechanisms} \\
 & & \text{for assignment of heterogeneous objects.}
\end{array}
$$

Goal:

$$
\begin{array}{lll}
m^* & = arg \max_{m \in \mathscr{M}} e(m) & \\
 & \text{s.t.} & \\
r_i(.) & \geq 0, \forall \theta \in \Theta \ \forall i \in N & \text{Individual Rationality} \\
\sum_i r_i(.) & \leq t & \text{Feasibility}
\end{array}
\qquad (6)
$$

- **Linear rebate function**: rebate function for an agent i is linear combination of bids of remaining $(n-1)$ agents

Our Contributions

- Impossibility of linear rebate functions with non-zero redistribution index
- Possibility of such functions in restricted settings
- Existence of non-linear rebate function that has non-zero efficiency
- HETERO, a non-linear rebate function
    - It satisfies desired properties and is optimal on worst case analysis

Sujit Gujar and Y Narahari, "**Redistribution Mechanisms for Assignment of Heterogeneous Objects**". *Journal of Artificial Intelligence Research (JAIR)*, 2011, Volume 41, pp: 131-154.

Impossibility of Linear Rebate Scheme

$\mathscr{M}_L$: Space of anonymous redistribution mechanisms in which rebate functions are linear.

Problem reduces to,

$$\boxed{\begin{array}{c} m^* = arg\max_{m \in \mathscr{M}_L} e(m) \\ \text{s.t.} \\ r_i(.) \geq 0, \forall\, \theta \in \Theta\ \forall i \in N \\ \sum_i r_i(.) \leq t \end{array}} \quad (7)$$

- For this setting, we prove, $e(m^*) = 0$

Theorem

If a mechanism has to be feasible and individually rational, there **does not exist** *a* **linear** *rebate function which satisfies all the following properties,*

- *DSIC*
- *deterministic*
- *anonymous*
- *guarantees non-zero redistribution in worst case*

[**?**] "Redistribution Mechanisms for Assignment of Heterogeneous Objects", Sujit Gujar and Y Narahari, *Journal of Artificial Intelligence Research, (JAIR), Vol 41, 131-154 (2011)* .

Sketch of the Proof

Step1 Define Ordering of the agents based on bids ($\succcurlyeq$)

Step2 We prove,

Theorem

Any deterministic, anonymous rebate function f is DSIC iff,

$$r_i = f(v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n) \;\; \forall\, i \in N \tag{8}$$

where, $v_1 \succcurlyeq v_2 \succcurlyeq \ldots \succcurlyeq v_n$.

Step3

$$\begin{aligned} r_i &= f(v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n) \\ &= (c_0, e_p) + (c_1, v_1) + \ldots + (c_{n-1}, v_n) \end{aligned}$$

Sketch of the Proof of Theorem Continued

Step 4 Show that, $c_0, c_1, \ldots, c_{p+1} = 0$

Step 5 Construct type profile such that, VCG Payment is non zero. However, the rebate to each of the agents is zero.

Sketch of the Proof of Theorem Continued

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How do we get around this impossibility result?

Route 1 Restrict the domain of types

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How do we get around this impossibility result?

Route 1 Restrict the domain of types

Route 2 Explore non-linear rebate functions

Route 1: Restrict the Domain of Types

Restrict the Domain of Types: An Example of Scaling Based Valuation

Motivation:

- Consider the website `somefreeads.com`
- There are p slots available for advertisements, and n agents interested in displaying their ads

Restrict the Domain of Types: An Example of Scaling Based Valuation

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Definition

We say the valuations of the agents have **scaling based relationship** if there exist positive real numbers $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_p > 0$ such that, for each agent $i \in N$, the valuation for object j, say θ_{ij}, is of the form $\theta_{ij} = \alpha_j v_i$, where $v_i \in \mathbb{R}_+$ is a private signal observed by agent i.

Proposed Mechanism

- The agents submit their bids.
- The bids are sorted in decreasing order.
- The highest bidder will be allotted the first object, the second highest bidder will be allotted the second object, and so on.
- Agent i will pay $t_i - r_i$, where t_i is the Clarke payment and r_i is the rebate.

$$t_i = \sum_{j=i}^{p} (\alpha_j - \alpha_{j+1}) v_{j+1}$$

Proposed Mechanism

- Let agent i's rebate be,

$$r_i = c_2 v_2 + \ldots + c_{i-1} v_{i-1} + c_i v_{i+1} + \ldots + c_{n-1} v_n$$

- Define $\beta_1 = \alpha_1 - \alpha_2$, and for $i = 2, \ldots, p$, let $\beta_i = i(\alpha_i - \alpha_{i+1}) + \beta_{i-1}$.
- Define $x_j = \sum_{i=2}^{j} c_i$ for $j = 2, \ldots, n-1$.

$$\boxed{\begin{array}{c} \text{maximize } e \\ s.t. \\ e\beta_1 \leq (n-2)x_2 \leq \beta_1 \\ e\beta_{i-1} \leq ix_{i-1} + (n-i)x_i \leq \beta_{i-1} \quad i = 3, \ldots, p \\ e\beta_p \leq ix_{i-1} + (n-i)x_i \leq \beta_p \quad i = p+1, \ldots, n-1 \\ e\beta_p \leq nx_{n-1} \leq \beta_p \\ x_i \geq 0 \ \ \forall i = 2, \ldots, n-1 \end{array}} \qquad (9)$$

The discussion above can be summarized as the following theorem.[5]

Theorem

When the valuations of the agents have scaling based relationship, for any p and $n > p + 1$, the linear redistribution mechanism obtained by solving LP (9) is worst case optimal among all Groves redistribution mechanisms that are feasible, individually rational, deterministic, and anonymous.

[5] **[?]** "Redistribution Mechanisms for Assignment of Heterogeneous Objects", Sujit Gujar and Y Narahari, *Journal of Artificial Intelligence Research, (JAIR), Vol 41, 131-154 (2011)*

Route 2: Explore Non-Linear Rebate Functions

BAILEY Mechanism

We apply Bailey [5][6] mechanism to these settings,

$$r_i^B = \frac{1}{n} t^{-i}$$

[6] Martin J Bailey. The demand revealing process: To distribute the surplus. Public Choice, 91(2):107-26, April 1997.

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Proposition

The BAILEY redistribution scheme,

- *is feasible*
- *individually rational and*
- *non-zero fraction of VCG surplus is always redistributed when $n > 2p$*

[6] Martin J Bailey. The demand revealing process: To distribute the surplus. Public Choice, 91(2):107-26, April 1997.

HETERO (Our Proposed Mechanism)

- $t^{-i,k}$: average payment received when agent i is absent along with k other agents

[7] Sujit Gujar and Y Narahari, "Redistribution of VCG payments in assignment of heterogeneous objects". In Proceedings of 4^{th} Workshop on Internet and Network Economics, WINE 2008. pg 438-445.

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- We propose [7]

$$r_i^{\mathrm{H}} = \alpha_1 t^{-i} + \sum_{k=2}^{k=L} \alpha_k t^{-i,k-1} \quad (10)$$

[7] Sujit Gujar and Y Narahari, "Redistribution of VCG payments in assignment of heterogeneous objects". In Proceedings of 4^{th} Workshop on Internet and Network Economics, WINE 2008. pg 438-445.

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where, $L = n - p - 1$ and for $i = p + 1 \rightarrow n - 1$

$$c_i = \sum_{k=0}^{n-i-1} \alpha_{L-k} \times \frac{\binom{i-1}{p}\binom{n-i-1}{k}}{\binom{n-1}{p+1+k}} \quad (11)$$

[7] Sujit Gujar and Y Narahari, "Redistribution of VCG payments in assignment of heterogeneous objects". In Proceedings of 4^{th} Workshop on Internet and Network Economics, WINE 2008. pg 438-445.

- α's are given by,

$$\alpha_i = \frac{(-1)^{(i+1)}(L-i)!p!}{(n-i)!}\chi\sum_{j=0}^{L-i}\left\{\binom{i+j-1}{j}\sum_{l=p+i+j}^{n-1}\binom{n-1}{l}\right\}; \quad i = 1, 2, \ldots, L \tag{12}$$

where, χ is given by, $\chi = \frac{(n-p)\binom{n-1}{p-1}}{\sum_{j=p}^{n-1}\binom{n-1}{j}}$

- HETERO agrees with the WCO mechanism when objects are homogeneous

How Does HETERO Works?

Our Conjecture

The proposed scheme, HETERO, is individually rational, feasible and worst case optimal.

[8] M Guo and V Conitzer, "Worst-case optimal redistribution of VCG payments". In EC'07: Proceedings of the 8^{th} ACM conference on Electronic Commerce, EC, pages 30-39, New York, NY, USA, 2007. ACM.

How Does HETERO Works?

Our Conjecture

The proposed scheme, HETERO, is individually rational, feasible and worst case optimal.

Our Conjecture is based on the result of Guo and Conitzer [8] :

Theorem 1

For any $x_1 \geq x_2 \geq \ldots x_n \geq 0$,

$$a_1 x_1 + a_2 x_2 + \ldots a_n x_n \geq 0 \;\; \textit{iff} \;\; \sum_{i=1}^{j} a_i \geq 0 \;\; \forall j = 1, 2 \ldots, n.$$

[8] M Guo and V Conitzer, "Worst-case optimal redistribution of VCG payments". In EC'07: Proceedings of the 8^{th} ACM conference on Electronic Commerce, EC, pages 30-39, New York, NY, USA, 2007. ACM.

How Does HETERO Works?

1. Define, $\Gamma_1 = t^{-i}$, $\Gamma_j = t^{-i,j-1}$, $j = 2, \ldots, L$
2. Rebate function for agent i,

$$r = \sum_j \alpha_j \Gamma_j$$

3. Note, $\Gamma_1 \geq \Gamma_2 \geq \ldots \geq \Gamma_L \geq 0$
4. For $p = 2$, $n = 4, 5, 6$; $p = 3$, $n = 5, 6, 7$; individual rationality follows from Theorem 1
5. If $\sum_{i=1}^{j} \alpha_i \geq 0 \; \forall \, j = 1 \rightarrow L$, individual rationality would follow from Theorem 1
6. Γ_j's are related
7. α_j's give appropriate weights to the combinations when a particular agent is absent in the system along with $j - 1$ agents

Experiments and Empirical Evidence

Setup 1

- $p = 2, n = 5, 6, \ldots, 14,$ $\quad$ # Experiments 200, 000
- $p = 3, n = 7, 8, \ldots, 14,$ $\quad$ # Experiments 40, 000
- $p = 4, n = 9, 10, \ldots, 14,$ $\quad$ # Experiments 40, 000

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Setup 2

Assume all the agents have binary valuations on each of these objects

- $p = 2, n = 5, 6, \ldots, 12$

Enumerate all possible bids.

HETERO is individually rational, feasible, and worst case optimal

HETERO

Our Conjecture

The proposed scheme, HETERO, is individually rational, worst case optimal.

HETERO

Our Conjecture

The proposed scheme, HETERO, is individually rational, worst case optimal.

- Guo [9] showed that HETERO is individually rational and worst case optimal for unit demand case.
- [9] also conjectured that it is a worst case optimal for any general combinatorial auctions with gross substitutes

HETERO: Proof

- $r_i^H = \sum_{j=p+1}^{n} \gamma_j R(N,j)$
- $R(N,0)$: VCVG Surplus
- $R(N,j)$ recursive definition in terms of $R(N-a, j-1)$
- $R(N,0) \geq R(N,1) \geq \ldots R(N, n-p-1) \geq 0$
- Invoke Theorem 1 to prove IR, Feasibility and Optimalaity of HETERO

We have Seen

- Need to Redistribution Mechanisms

× No linear rebate function can guarantee non-zero redistribution index for heterogeneous objects case

- Two ways to get around this
  - Restrict the domain of types
  - Non-linear rebate functions

- Scaling Based Relationship

- HETERO,

√ HETERO agrees with Moulin scheme in case of homogeneous objects

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Questions?

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Thank You!!!