

# Information Aggregation in Prediction Markets

Rahul Sami

University of Michigan School of Information

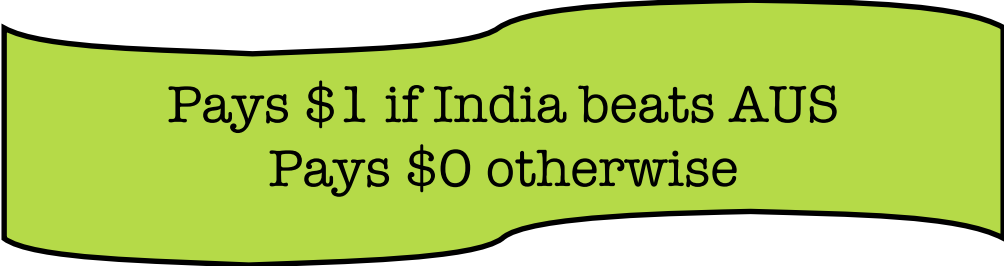
Current affiliation: Google, Bangalore

NMI Workshop on Game Theory and Mechanism  
Design

# Talk Goals

- Introduction to scoring rule and market scoring rule mechanisms
- Theoretical analysis of strategies
  - with connections to information theory
- Small peek into experimental methods

# Information/Prediction Markets



Pays \$1 if India beats AUS  
Pays \$0 otherwise

Markets designed to aggregate traders' information.

- Issue securities with value contingent on future event.
- Trading price is taken as a prediction of future value.
- Once event occurs, security is cashed out for money

# Example:Iowa Electronic Markets

iemweb.biz.uiowa.edu/quotes/Pres12\_quotes.html

isited ▾ Getting Started Latest Headlines ▾ Ann Arbor - Mic...

[Markets](#) ▫ [2012 Presidential](#) ▫ [2012 Presidential Data](#) ▫ [2012 Presidential Quotes](#)

## Market Quotes: Pres12\_VS 2012 Presidential Election Vote Share Market.

Quotes current as of 14:15:04 CST, Monday, October 31, 2011.

Symbol	Bid	Ask	Last	Low	High	Average
UDEM12_VS	0.478	0.497	0.499	---	---	---
UREP12_VS	0.511	0.518	0.510	---	---	---

| [Prospectus](#) | [Price History](#) | [Graph](#) |

## Market Quotes: Pres12\_WTA 2012 Presidential Election Winner-Take-All Market.

Quotes current as of 14:15:04 CST, Monday, October 31, 2011.

Symbol	Bid	Ask	Last	Low	High	Average
DEM12_WTA	0.506	0.513	0.510	0.510	0.510	0.510
REP12_WTA	0.491	0.493	0.492	0.490	0.492	0.490

| [Prospectus](#) | [Price History](#) | [Graph](#) |

# Markets aggregate information

The screenshot shows the Iowa Electronic Markets (IEM) website. At the top left is the IEM logo and the text "Iowa Electronic Markets". At the top right is the logo for "THE UNIVERSITY OF IOWA HENRY B. TIPPIE COLLEGE OF BUSINESS". A red navigation bar contains three links: "Login and Trade", "Open an Account", and "Current Market Quotes". Below the navigation bar, the date "October 30, 2005" is displayed. On the left side, there is a blue sidebar with several underlined links: "About the IEM", "FAQ", "Current Markets", "Instructor Resources", "Account Maintenance", and "Trader's". The main content area features a paragraph of text: "The Iowa Electronic Markets are real-money futures markets in which contract payoffs depend on economic and political events such as elections. These markets are operated by faculty at the University of Iowa Tippie College of Business as part of our research and teaching mission. We invite you to join us in this mission." Below this paragraph is a red arrow pointing to the underlined link "Political Markets". To the right of the main text is a rounded rectangular box titled "Announcements" containing the text: "Trading is currently open in our market based on the monetary policy decisions of the Federal Open Market Committee regarding the federal funds target rate."

The Iowa Electronic Market predicts election outcomes better than opinion polls [Forsythe *et al.* '99].

# Markets aggregate information

The screenshot shows the Tradesports.com website interface. At the top left is the logo for Tradesports.com, described as 'A Trade Exchange Network Company' with a 'Start Trading' button. To the right are fields for 'Username:' and 'Password:' with a 'Login' button and a 'Join Now' button. Below the header is a navigation menu with links for 'Home', 'Join', 'Trade Here', 'Rules & FAQs', 'About Us', 'Forum', and 'Feedback'. The main content area displays a dropdown menu for '2006 Stanley Cup Outright Winner' and a timestamp of '6:51:44PM GMT' with a 'Refresh' button. A table shows market data for four NHL teams, with columns for 'Contract', 'BQty', 'Bid', 'Offer', 'AQty', 'Last', 'Vol', and 'Chg'.

		Best to Sell		Best to Buy				
Contract		BQty	Bid	Offer	AQty	Last	Vol	Chg
Trade	NHL.FLYERS	1	11.0	13.0	200	13.5	988	0
Trade	NHL.SENATORS	192	14.0	16.9	1	14.0	1251	-1.0
Trade	NHL.REDWINGS	169	13.0	14.5	1	13.0	1183	+2.0
Trade	NHL.AVALANCHE	10	3.3	3.9	25	3.9	492	0

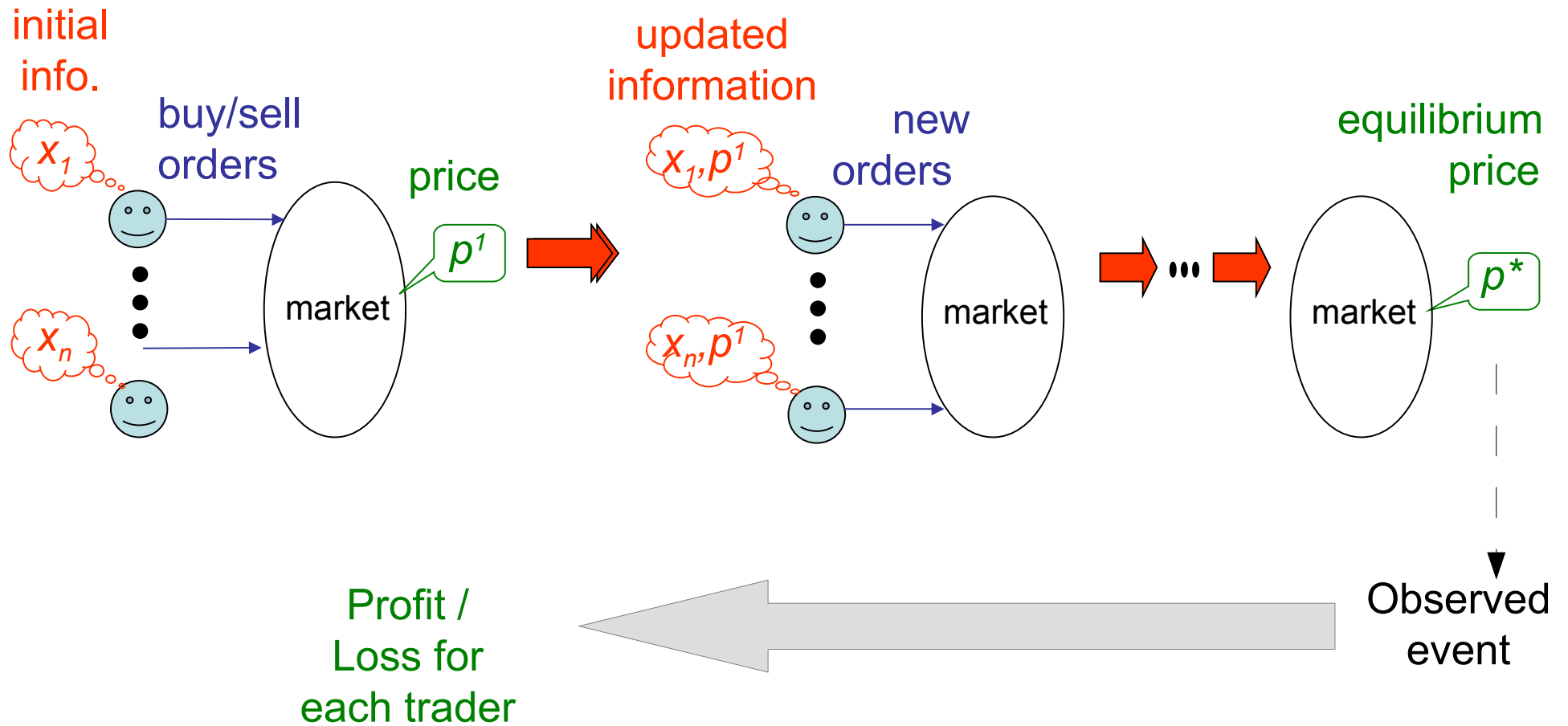
Sports betting markets provide unbiased forecasts of game outcomes [Gandar *et al.* '98; Debnath *et al.* '03]

# Markets Designed for Aggregation

The screenshot shows the Hollywood Stock Exchange (HSX) website. The top navigation bar includes links for 'home', 'movies', 'H\$hop', 'login', and 'join'. The main header reads 'HOLLYWOOD STOCK EXCHANGE'. Below this, there are two columns: 'MOVIESTOCKS' and 'STARBONDS'. The 'MOVIESTOCKS' column features a large graphic for 'Final Destination 3' (FDES3) with a price of H\$164.09 and a change of 3.78. Below it are other movie stocks: 'The Chronicles of Fantasic Four' (H\$64.32, +2.00), 'Sin City 2: A Jarhead' (H\$55.80, -2.66), and 'Chicken Little' (H\$89.73, -3.50). The 'STARBONDS' column features a large graphic for 'Michael Caine' (MCAIN) with a price of H\$79.68 and a change of 4.00. Below it are other actor stocks: 'Johnny Depp' (H\$119.89, +2.00), 'Morgan Freeman' (H\$74.50, +2.00), 'Mike Newell' (H\$91.77, -2.00), and 'Michael Madsen' (H\$71.36, -2.00). On the left side of the page, there is a 'sign-up [learn more]' button and a small HSX logo with some market data.

Markets sometimes deployed primarily for information aggregation (e.g., IEM, Hollywood Stock Exchange)

# Market as Incentive Mechanism



$$\text{equilibrium price } p^* \Leftrightarrow f(x_1, x_2, \dots, x_n)$$

**Goal: Profit incentive should induce optimal aggregation**



# Single Forecaster Incentives

Decision Maker

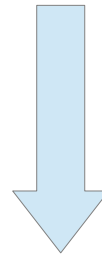
*What is the probability of rain tomorrow?*

q

Expert



RAIN or DRY



Reward

# Scoring rules

A *scoring rule* is a rule that is used to compute the reward for a forecaster.

- Depends on the forecast probability of rain  $q$
- Must also depend on actual outcome
- For this example, scoring rule consists of two functions:

$$S_{RAIN}(q), \quad S_{DRY}(q)$$

# Example: Linear scoring rule

- If you say “It will rain with probability  $p$ ” and it rains  $\Rightarrow$  your reward is  $\$p$
- If you say “it will rain with probability  $p$ ” and it is dry  $\Rightarrow$  your reward is  $\$(1-p)$

If you think the probability is 80% of rain (and say so), what is your expected reward?

# Proper scoring rule

*Proper scoring rules satisfy the following property:*

*If a forecaster believes the probability of an event is  $p$ , her expected reward is maximized by reporting  $q=p$ .*

There are several well-known proper scoring rules:

- *Quadratic Scoring Rule [Brier 1952]*
  - *Logarithmic Scoring Rule [Good 1950]*
  - *Spherical Scoring Rule*
- x Linear scoring rule is *not* proper

# Logarithmic Scoring Rule

$$S_{RAIN}(q) = \log q$$

$$S_{DRY}(q) = \log(1-q)$$

Log scoring rule is a proper scoring rule:

# Logarithmic Scoring Rule

$$S_{RAIN}(q) = \log q$$

$$S_{DRY}(q) = \log(1-q)$$

Log scoring rule is a proper scoring rule:

$$\begin{aligned} E_p [ \text{Score}(q) ] &= p \log q + (1-p) \log (1-q) \\ &= p \log p + (1-p) \log p + \\ &\quad p \log (q/p) + (1-p) \log [(1-q)/(1-p)] \end{aligned}$$

# Logarithmic Scoring Rule

$$S_{RAIN}(q) = \log q$$

$$S_{DRY}(q) = \log(1-q)$$

Log scoring rule is a proper scoring rule:

$$\begin{aligned} E_p [ \text{Score}(q) ] &= p \log q + (1-p) \log (1-q) \\ &= p \log p + (1-p) \log p + \\ &\quad p \log (q/p) + (1-p) \log [(1-q)/(1-p)] \\ &= -H(p) - D(p \parallel q) \end{aligned}$$

Entropy    KL-divergence

Optimal  $q$  :  $q=p \Rightarrow$  Expected score =  $-H(p)$

# Logarithmic Scoring Rule: Variations

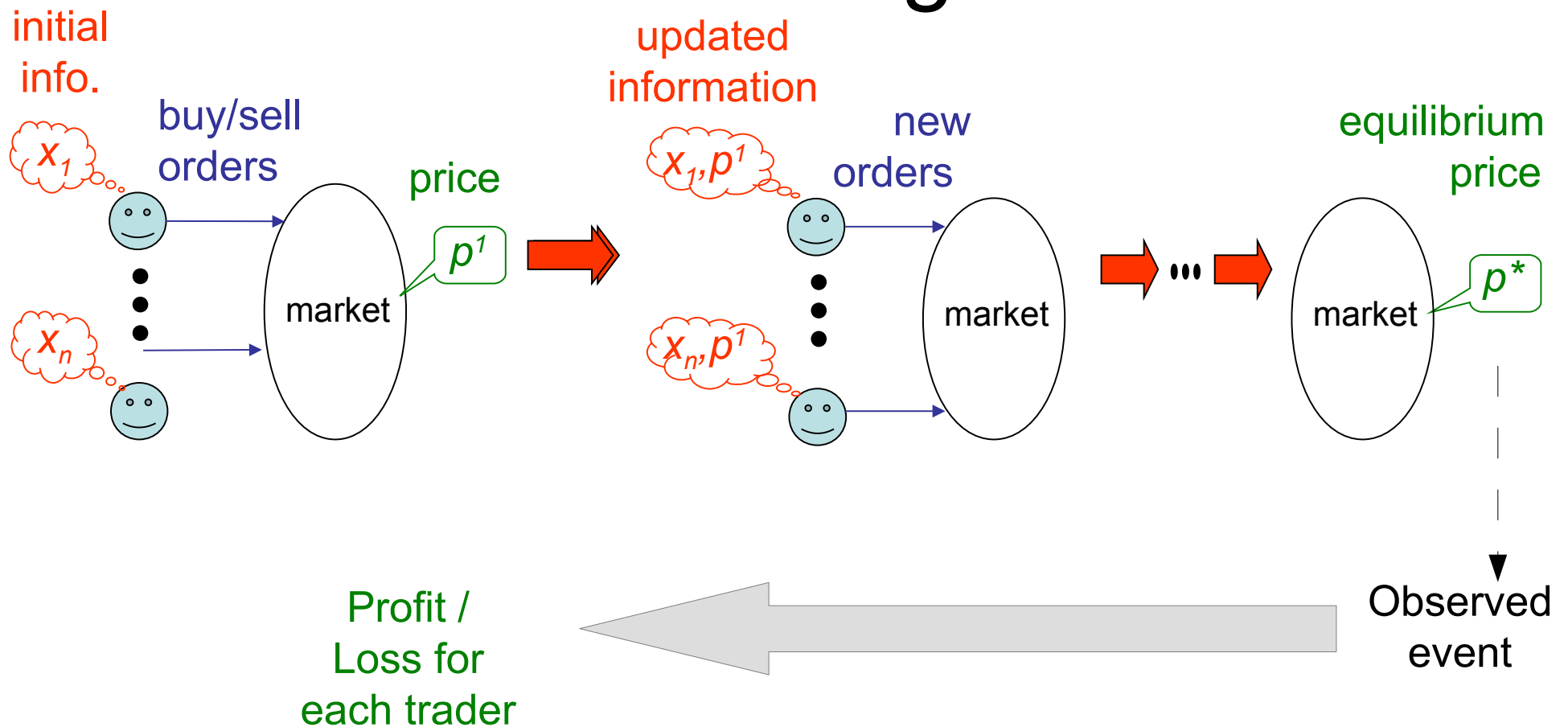
$$S_{RAIN}(q) = a + b \log q$$

$$S_{DRY}(q) = a + b \log(1-q)$$

Constants  $a, b$  control scale and absolute value of rewards while retaining strategic properties.



# Multiple Forecasters: Market Setting

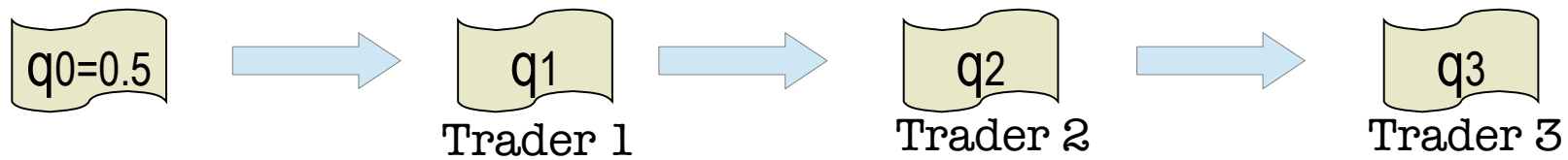


$$\text{equilibrium price } p^* \Leftrightarrow f(x_1, x_2, \dots, x_n)$$

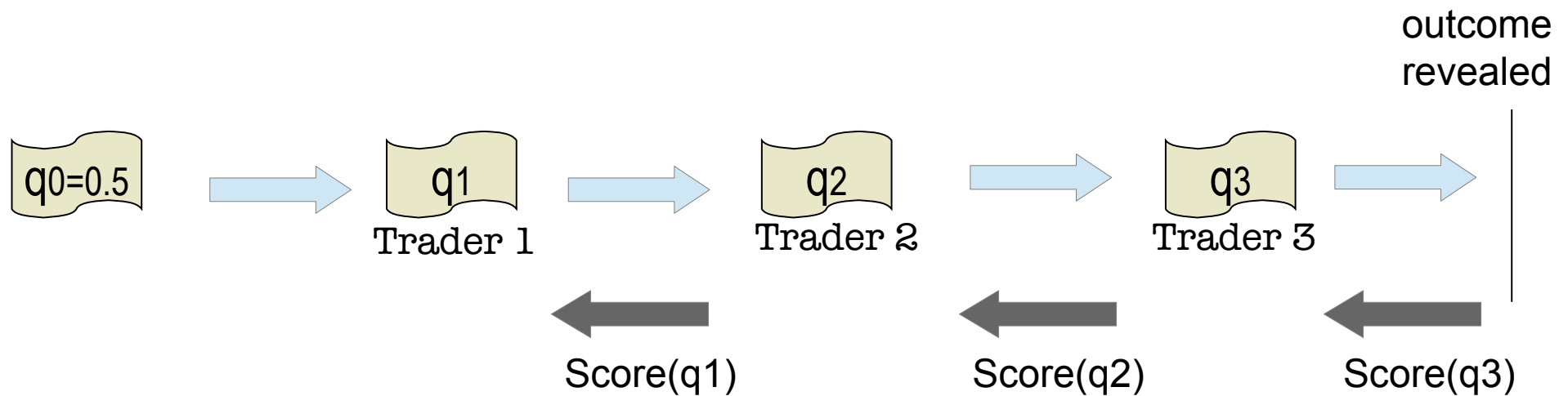
**Goal: Profit incentive should induce optimal aggregation** 17

# Market Scoring Rules [Hanson 03]

Market based on trading scoring rules

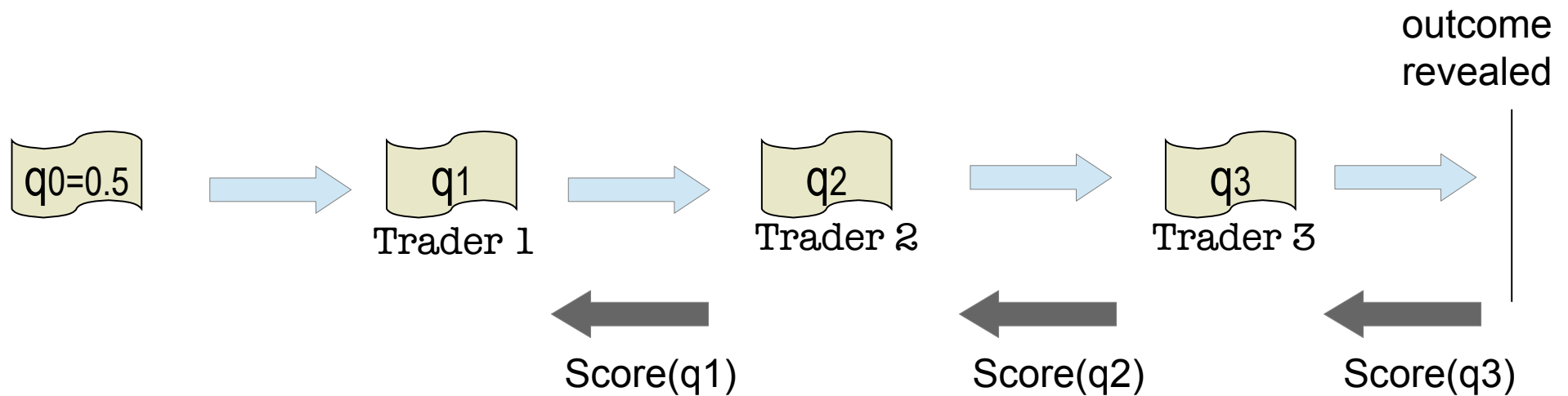


# Market Scoring Rule Payoffs



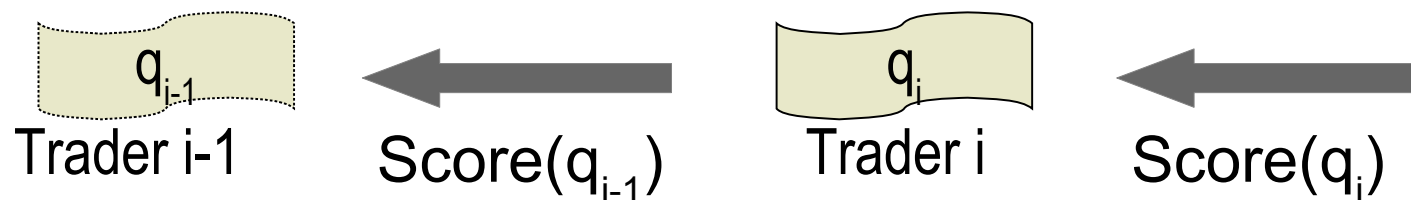
- Market maker rewards last trader
- Each trader pays previous trader's reward

# Market Scoring Rule Payoffs



- Market maker rewards last trader
- Each trader pays previous trader's reward

# Strategies with Market Scoring Rules



- Profit of trader  $i$ :

$$\text{Profit}(i) = \text{Score}(q_i) - \text{Score}(q_{i-1})$$
$$E(\text{Profit}(i)) = E(\text{Score}(q_i)) - E(\text{Score}(q_{i-1}))$$

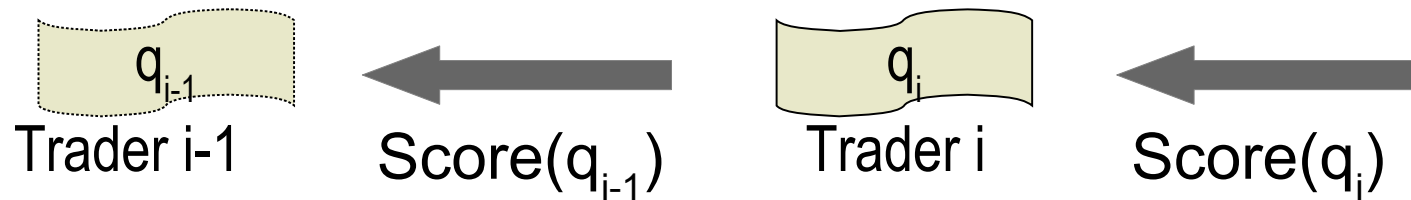
- Truthful reporting is “myopically” optimal strategy.
  - ie., if you rule out misleading other traders to make a long-term profit

# Market maker's gain or loss

- Market operator can specify a maximum “endowment” for the market maker
- This controls:
  - how much MM can win or lose
  - how sensitive instantaneous prices are to one unit bought/sold
  - Equivalently: the constant multiplier ‘b’ in the underlying scoring rule:

$$S_{YES}(q) = b \log p \quad ; \quad S_{NO}(q) = b \log(1-p)$$

# Information-theoretic characterization of profit



- Profit of trader  $i$ :

$$\begin{aligned}\text{Profit}(i) &= \text{Score}(q_i) - \text{Score}(q_{i-1}) \\ E(\text{Profit}(i)) &= E[\text{Score}(q_i)] - E[\text{Score}(q_{i-1})]\end{aligned}$$

- Assuming all reports are truthful:
  - Expected Profit of trader  $i =$

$$b[ H(\text{Event} | x_1, x_2, \dots, x_{i-1}) - H(\text{Event} | x_1, x_2, \dots, x_i) ]$$

# Alternative view: automated market maker

The market scoring rule can also be viewed as an automated market maker

- “Instantaneous prices” are set based on current probability
- For log-MSR, if M shares on outcome X and N shares on outcome Y have been sold,

*instantaneous price of X =*

$$\frac{e^{M/b}}{e^{M/b} + e^{N/b}}$$

- updated for every little additional unit bought or sold
- Payoff of outcome that happens = 1
- Trader buys/sells until price exactly matches her expected value for the security

Mathematically equivalent to previous description!



# Market Scoring Rules : Summary of Basic Properties

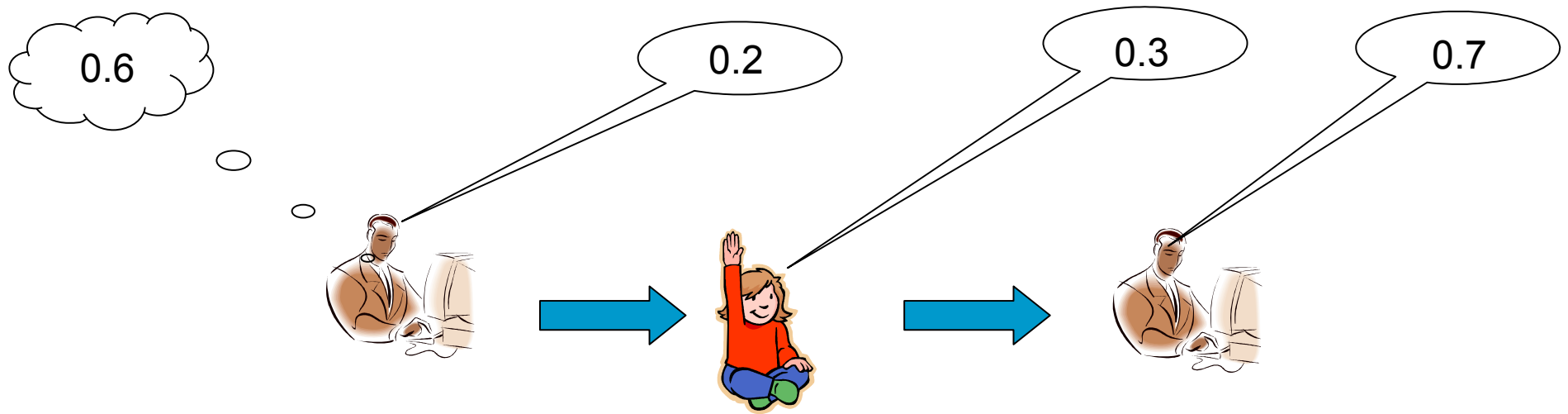
[Hanson03]

- Truth-revealing is strategically optimal (myopically)
- Can be implemented as a price-setting market maker
- Market-maker's loss / subsidy is bounded (regardless of number of traders)
- Expected profit connected to entropy

Long-term strategies?

# Long-term Incentives to be Untruthful

Is it ever profitable to bluff and correct?



# Motivating example

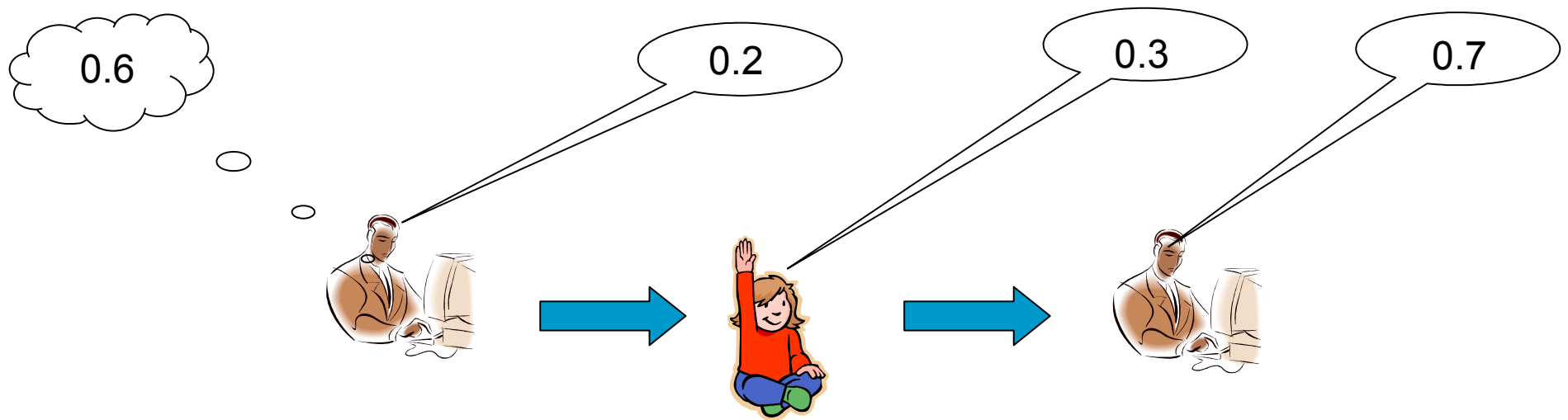
- Trader1 information:
  - '1' : IND focus on batting (with prior 0.49),
  - '0' IND focus on bowling (with prior 0.51)
- Trader2 information:
  - '1' : AUS focus on batting (with prior 0.49),
  - '0' : AUS focus on bowling (with prior 0.51)
- True outcome:  $XOR(x_1, x_2)$

# Motivating example

- True outcome:  $XOR(x_1, x_2)$ , prior = 0.5
- If Trader 1 sees '1':
  - Truthful:
    - Move price 0.5  $\rightarrow$  0.49
    - Trader 2 moves from 0.49 to 1 or 0
  - Bluff:
    - Move price from 0.5  $\rightarrow$  0.51
    - Trader 2 moves to 0 or 1
    - Trader 1 flips price to 1 or 0!

# Single market: Is honest play optimal?

[Dimitrov, S. '07]



- Assumption: traders get **independent** signals
- Thm: Generically, honest play is not an equilibrium strategy

# A different model: conditional independence

[Chen, Reeves, Pennock, Hanson, Fortnow, Gonen '07]

- Truthful reporting is an equilibrium strategy!
  - Assume that signals are **conditionally independent**, conditioned on the (unknown) true value

# Resolving the different results:

[CDSRPHFG '10]:

- Critical factor: Are signals *substitutes* or *complements*?
  - Value of signal is reduction in entropy due to signal
  - May be different before/after knowing other signal
- Truthful reporting is an equilibrium in former case



# Handling Complementarity: Discounted Market Scoring Rule

- One solution: discount profits over time [DS'07]
  - Second round payoff is  $\delta b(\log q_2 - \log q_1), \dots$
- Bluffing still possible..
- But, market converges to the optimal price:
  - Thm: In any weak-perfect bayesian equilibrium, the distribution of prices  $p_t$  after  $t$  trades each satisfies:  $E [D(p^* || p_t)] < c\delta^{at}$

# What happens in the real world?

Lab experiments are a good first step at testing theory predictions

# Experiments: Effect of information structure and market form

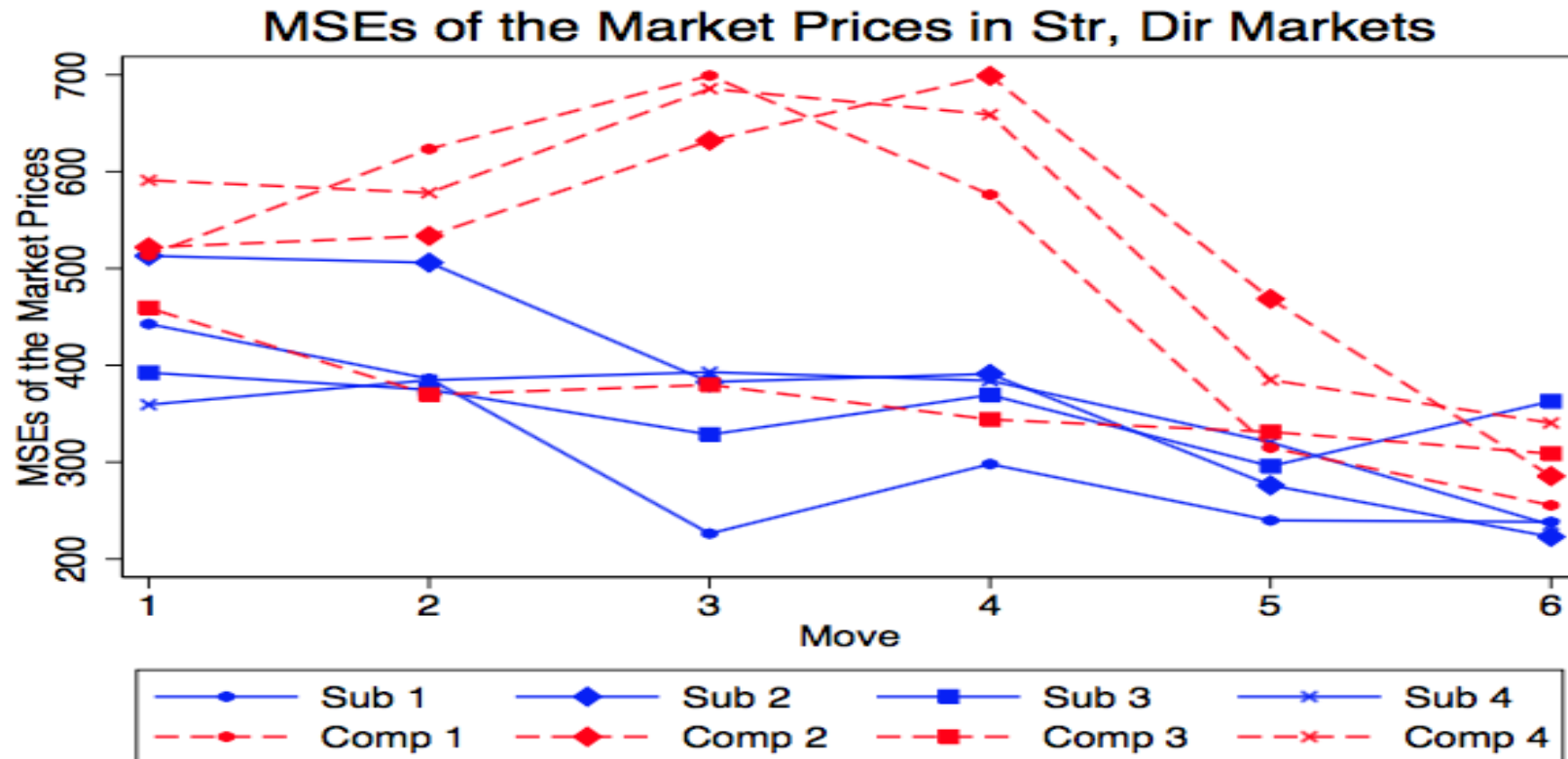
[Jian, S., 2010]



# Experiment design

- 2-player markets with repeated play
- 8 treatments
  - Independent (complements) vs. CI (substitute)
  - Interface variations: prediction vs. trading
  - Structured vs. Unstructured
- 4 sessions/treatment, 8 subjects/session
- Measure: Intermediate and Final price accuracy

# Results: Structured trading markets



- ✓ Result: Error in Complementary case after 2,4 rounds was significantly higher than in Substitutes case.

# Results: Unstructured trading markets

- × Result: Error in Complementary case after 2 trades was not significantly higher than in Substitutes case.

## Takeaways:

- Theoretical model was predictive when trading format exactly matched model
- .. but real-world natural trading is more complex for participants (and analysts!)
- Bluffing strategies were used in both complements and substitutes treatments (more in complements)
- Aside: Structured trading helps with more effective aggregation

# Conclusion

Prediction markets are an exciting class of mechanisms to study!

- Real-world applications and success stories
- Information-theoretic measures of value
- Rich strategic problems

A few directions for future work:

- Better modeling of real market microstructures
- More complete analysis of information settings and strategies
- Other market forms, scoring rules, etc.