Composition Games for distributed systems:
EU Grant games

Shay Kutten *
Ron Lavi *
Amitabh Trehan * $

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* Technion – Israel Institute of Technology
$ Hebrew University of Jerusalem/ Queen's University Belfast

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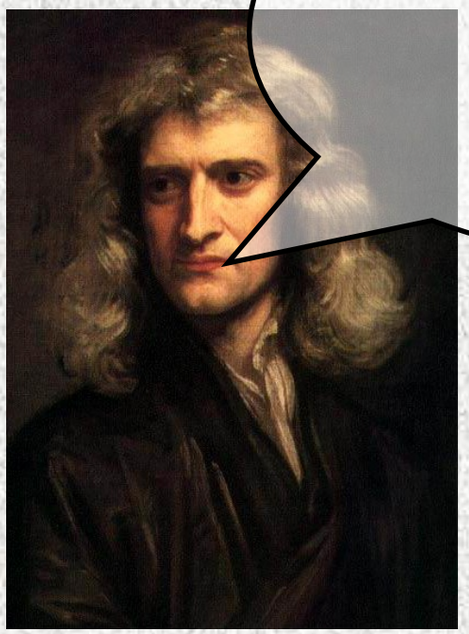
Brussels vs. the USA

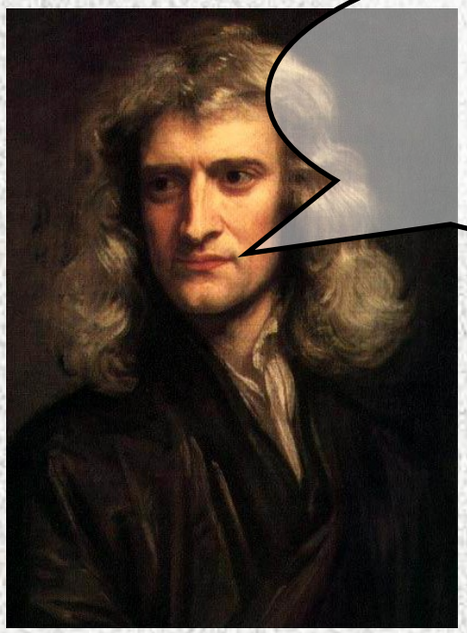
How to better compete with the USA?



I want a research grant!

Come with a bigger group!

Brussels vs. the USA

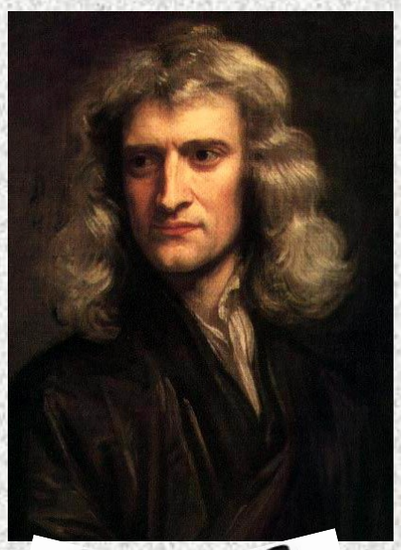


Yes sure!

If I have seen further than others, it is by standing upon the shoulders of giants.

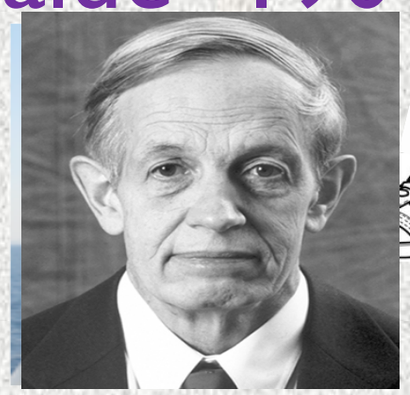
I want highest average group. Total above 300 (the "threshold")

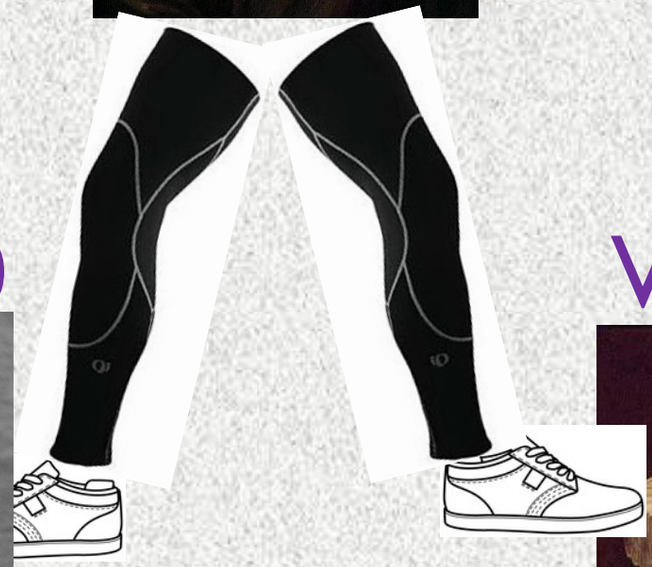
Assume the EU uses researchers' values to evaluate the quality of Isaac's group



Value=50

Value=190





Value=60

Total group value: 50+60+190; average: 100

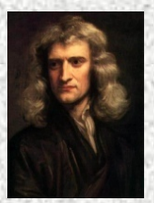
Want highest average.
Total above 300

Needed: a game for composing a winning group of the kind the agency wants

EU Grant Games:

Resulting group **not too small** (below threshold)
not too large (grant divided between many members)

Average: 100



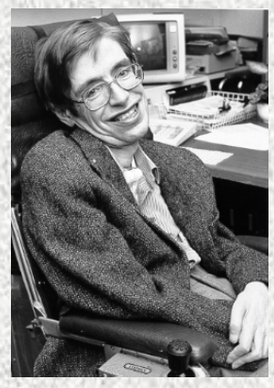
50

190

60

Not eligible

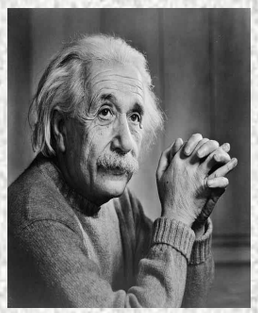
299



Winning group:

Average: 113

51



51

299

51

(large- maybe not a strong equilibrium)

Motivations (1)

"**Big problems e.g. …**

…how science budget should be allocated…or NSF budget among different areas? Given a subset of researchers, say we can estimate their impact… Given this oracle, can we allocate funds to people to maximize social welfare? Can we model people switching teams in second round or open bid systems for reallocating funds?

Q: Why doesn't NSF give $'s to 2 teams for the same project and get them to compete?

For some recent work, see the work of Shay Kutten, Ron Lavi and Amitabh Trehan." –

My slice of pizza" (blogspot), by S. Muthu MuthuKrishnan, July 4, 2011

Motivations (II)

".. 6) Collaborative projects are much easier to manage when it's 2-3 PI's (primary investigators) who are close collaborators than project involving a large number (say 7 or more) PI's. "

STOC grant-writing panel recap.

Theory Matters (wordpress) by Boaz Barak. June 12, 2013.

Motivations (III, IV)

- Composition of distributed systems of the "EU-grant type,"

  E.g. some P2P (**big enough** to have enough files to share, **small enough** to avoid congestion, law suit hazards…, etc.)

- Biological Ecosystems

Reminder:

EU-Grant Games:

Resulting group **not too small** (below threshold)
not too large (grant divided between many members)

Evaluating games

Nash Equilibrium: N.E.

No player profits by changing strategy.

A tuple of strategies $\ell_1, \ldots, \ell_n$ is a N.E. if

$u_i(\ell_1, \ldots, \ell_n) \geq u_i(\ell_1, \ell_{i-1}, \ell'_i, \ell_{i+1}, \ldots, \ell'_n)$

for every $i = 1, \ldots, n$

Strong (Nash) Equilibrium: S.E.

No group of players can jointly deviate and increase each of their utilities.

A tuple of strategies $\ell_1, \ldots, \ell_n$ is a S.E. if for any $\ell'_1, \ldots, \ell'_n \in L$ there exists a player i such that $\ell'_i \neq \ell_i$ and $u_i(\ell_1, \ldots, \ell_n) \geq u_i(\ell'_1, \ldots, \ell'_n)$

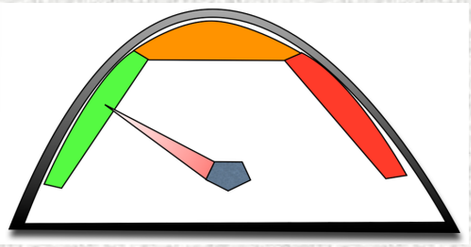
Evaluating games

Strong Price of Anarchy: SPoA

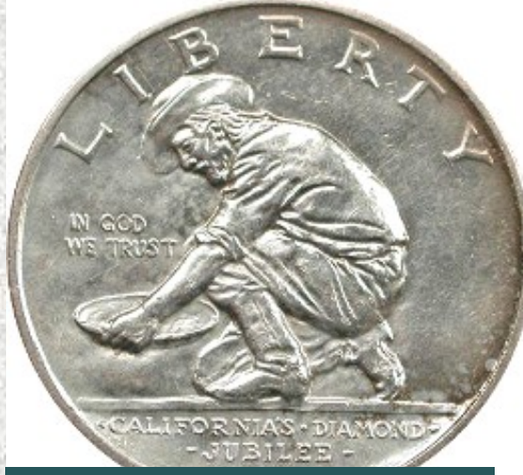
$$\text{SPoA} = \frac{\text{Social Optimum}}{\text{Worst Strong Equilibrium (S.E.)}}$$

$$= \frac{\text{average(`best' group)}}{\min(\text{average(winning group in S.E.)})}$$

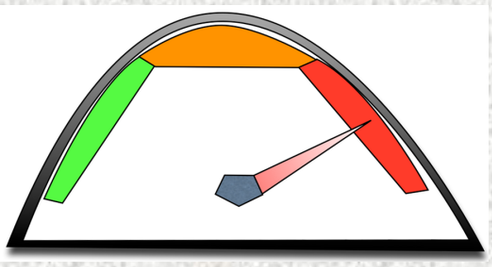
Some results:
The Difficulto-Meter



- Naive game where anybody may decide to join any group (**"easy" join**): unbounded **P**rice **o**f **A**narchy. (No strong equilibria)

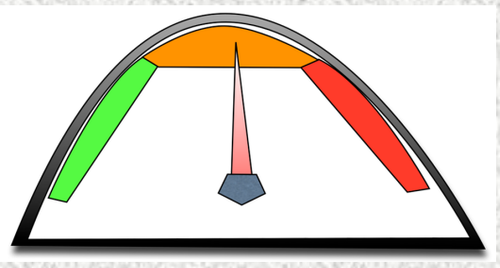


"Gold rush" game



"Hard" join game: Making the join to a group harder: better (but still high) SPoA.

"Consensual Consortium Composition" game



A game with SPoA tending to 1 **("medium" difficulty to join a group).**

"Israeli MAGNET grants" game

Further results

- We define a logical **network** of researchers (e.g. by collaboration).
- Assume that a grant winning group must be a connected component.
- Results: some dependencies of Strong Price of Anarchy on graph parameters (e.g. diameter)

EU Games: Basic Setup

- Grant: *M* Euros
- *n* researchers; researcher *i* has value v_i
- Threshold *T*;
  - $v_i < T$ for all i
  - Sum(v_i) > *T*, over all i

- Objective of the agency: Allot the Grant to the best 'consortium' according to the rules of the particular game

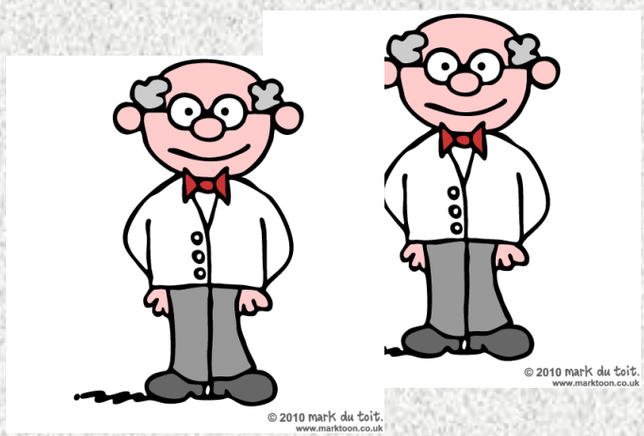
- Utility of players:
  - $U_i = 0$ if i is part of losing group
  - $U_i = M / y$ if i is part of winning group of size *y*

EU Games: Basic Setup

- Each researcher is part of exactly one consortium.

- The granting agency does not know value of researchers before hand, but it can verify the values when a 'grant proposal' is submitted.

We don't talk anymore!



- Not everybody collaborates with each other

Collaboration Networks

- The graph of collaboration where nodes are researchers and there is an edge between two researchers if they are willing to collaborate.
- *Feasible Eligible consortium:* Consortium with sum greater than T and induced subgraph in collobaration network is connected.

The Gold Rush Game

- Each researcher submits a separate proposal with some label ``consortium name'' from a finite set of labels.
- Researchers with same label belong to same consortium.
- *Eligible consortium:* a consortium with sum of values greater than *T*
- *Winner:* An eligible consortium with maximal average value

The Gold Rush Game

✧On a complete graph with distinct values, in every *Nash Equilibria (N.E.) either: a) no eligible consortium forms, or b) all researchers declare the same label.*

✧The price of anarchy of the gold-rush game is (arbitrarily close to) n/2

✧If there exists an eligible group which is a strict subset of the society, then there does not exist any strong equilibria (S.E.)

The Gold Rush Game

The Price of Anarchy of the gold-rush game is ~ n/2

Proof:

Let sum of highest two values be *V*

Social Opt (SO) = *V*/2

Avg(W) > *V*/*n* (assuming no contribution from others)

POA = SO/W ≃ n/2

The Gold Rush Game

If there exists an eligible group which is a strict subset of the society, then there does not exist any strong equilibria (S.E.)

Proof:

Every S.E. is a Nash Equilibrium.

a) Either the Nash have no eligible group. Thus, players can deviate to form an eligible group.

b) Or everybody is in the same group. Now, higher average eligible groups can deviate to form a smaller subset by declaring a new label.

The Gold Rush Game

Why no gold in the gold rush?

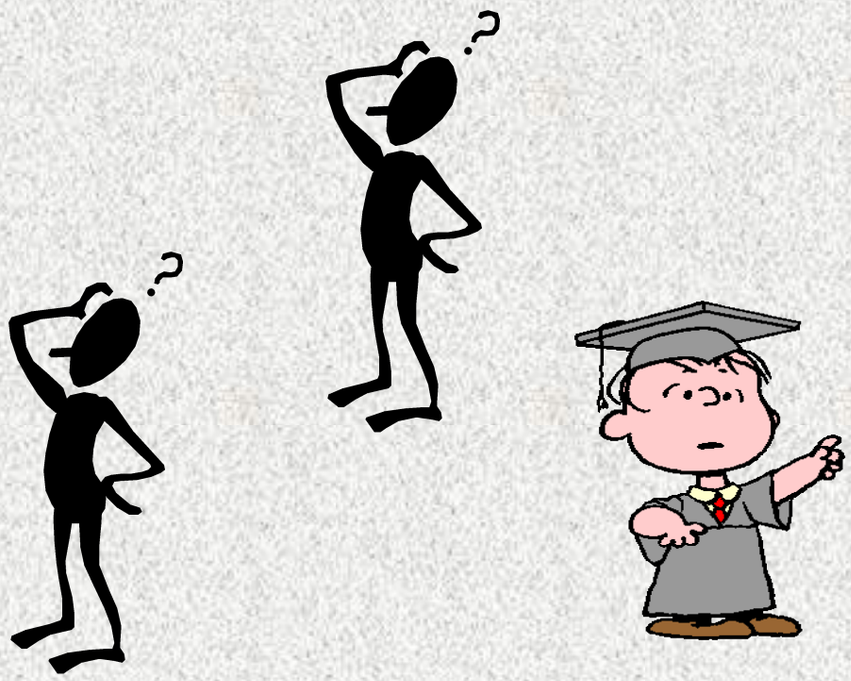
The Gold Rush Game

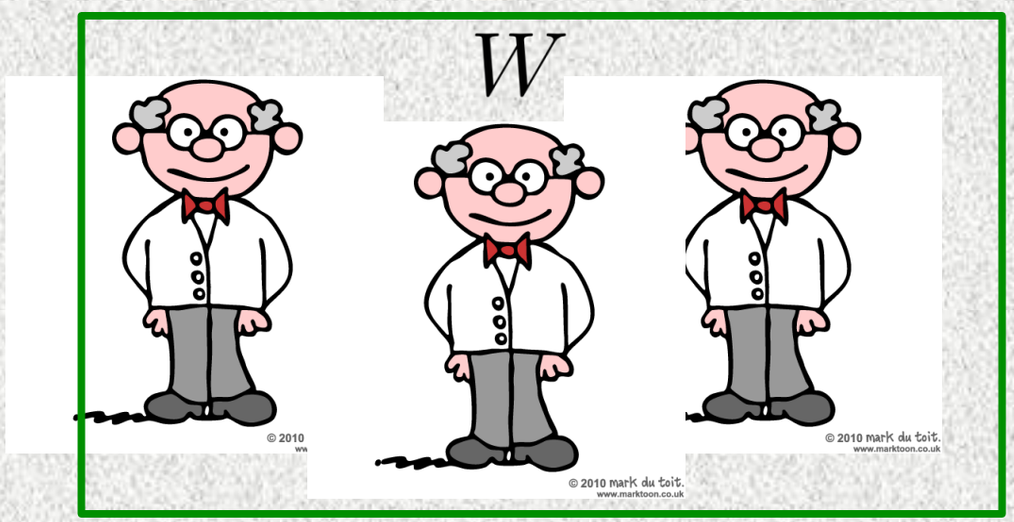
Why no gold in the gold rush?

Too easy to join

Consensual Consortium Composition (CCC)

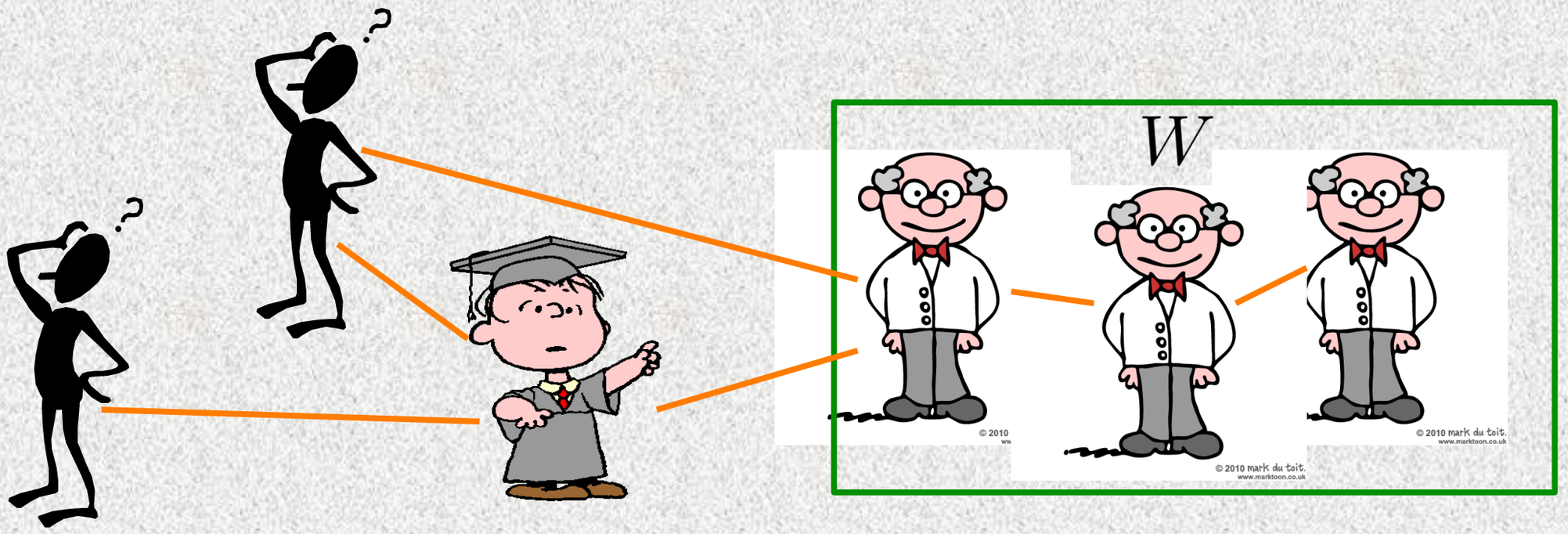
- ***Player strategies:*** A ``proposal'' with her value and list of researchers in consortium.
- ***Winner:*** eligible *consensual* group with highest average i.e. every consortium member approves everybody else explicitly.





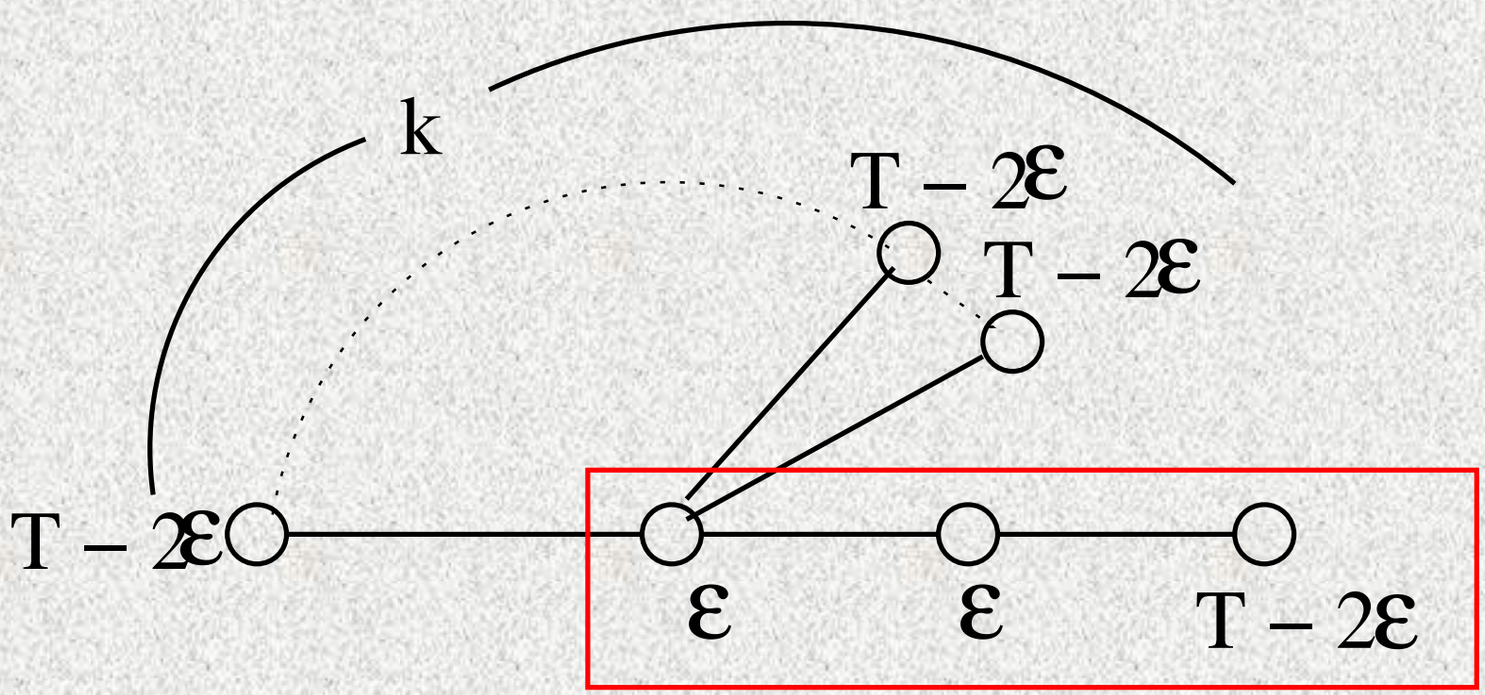
CCC with Collaboration Network (CCC-CN)

- ***Player strategies:*** A ``proposal'' with her value and list of researchers in consortium.
- ***Winner:*** feasible eligible *consensual* group with highest average i.e. every consortium member approves everybody else explicitly.



CCC with Collaboration Network (CCC-CN)

- Strong Equilibria always exists
- *SPOA* can be arbitrarily close to 3



CCC with Collaboration Network (CCC-CN)

- *SPOA*: Why so high?

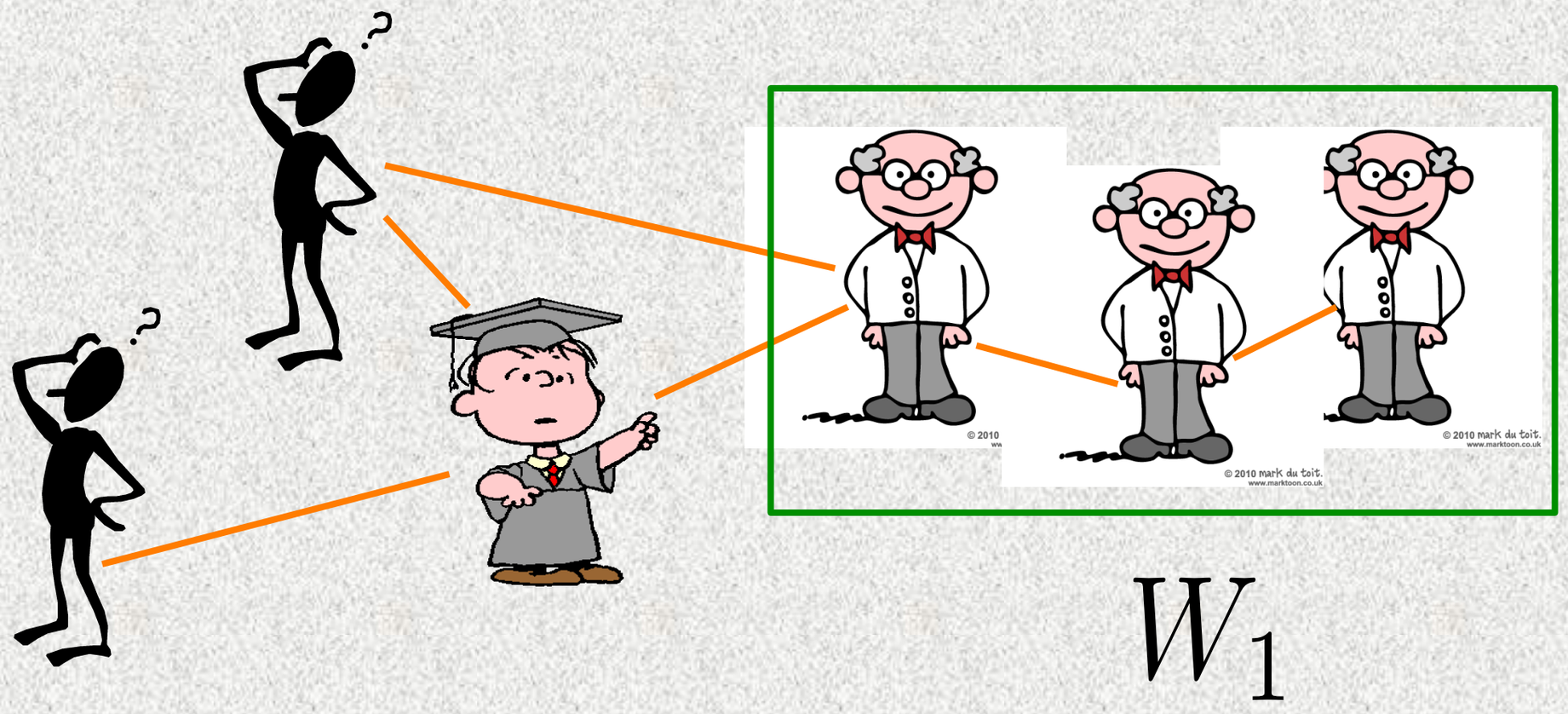
CCC with Collaboration Network (CCC-CN)

- *SPOA*: Why so high?

Too difficult to join?

MAGNET CCC game

- First application: Round 1: W_1= *consensual* group with highest average (up to here, this is CCC-CN).



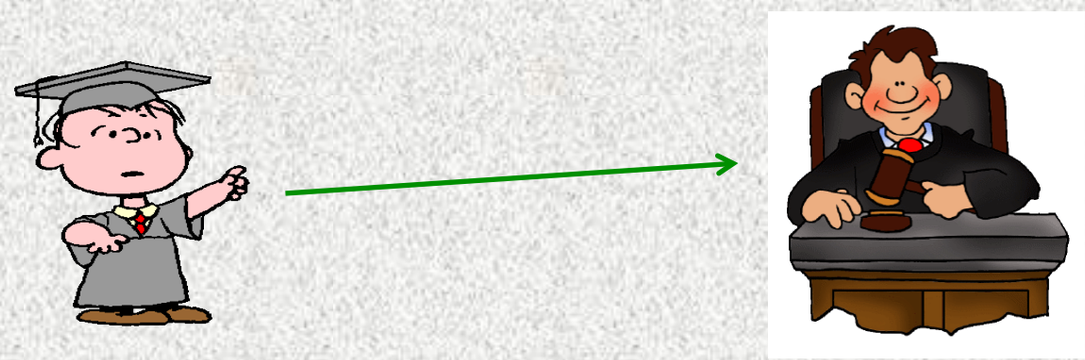
W_1

MAGNET CCC game

- Expansion to multiple rounds:

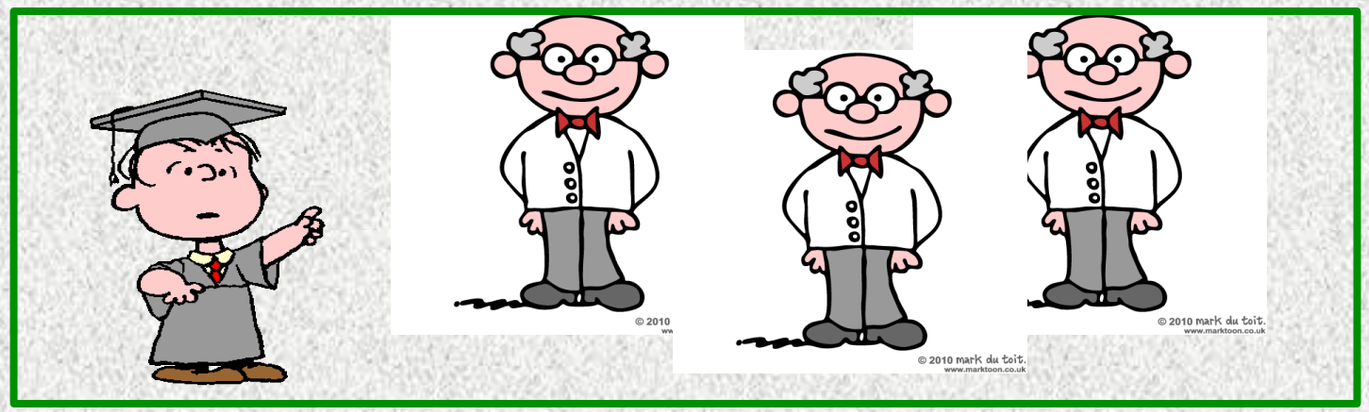
Round r: $W_r = W_{r-1} \cup X$ where X is an appealing consortium , W_r is connected and

$$average(W_{r-1} \cup X) > average(W_{r-1})$$

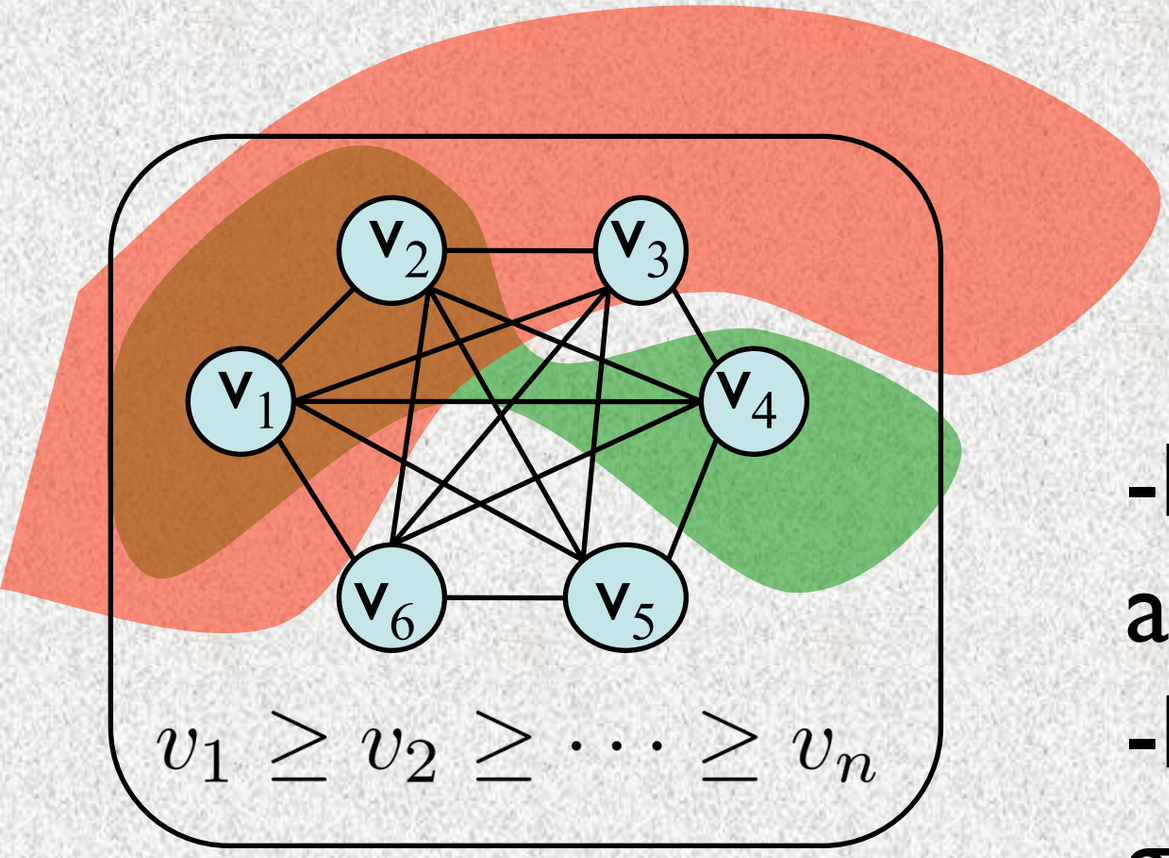


- Winner: (fixed point) $W_r = W_{r-1}$

W_r



SPoA for MAGNET game



Social opt (SO)

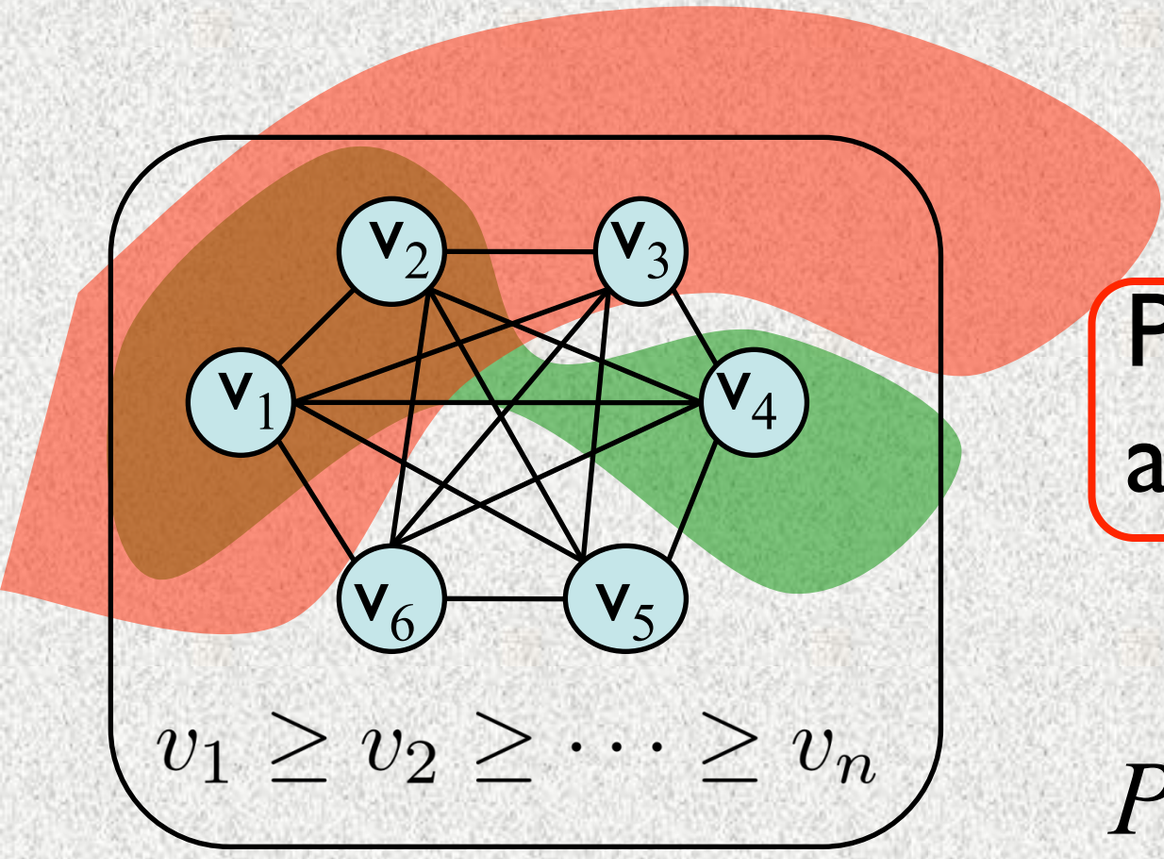
Winning group (W)

-Property 1: Intersection of SO and W is non-empty

-Property 2: Size of W is not greater than Size of SO

-Property 3: Average of W is greater than Average of (SO minus W)

SPoA for MAGNET game



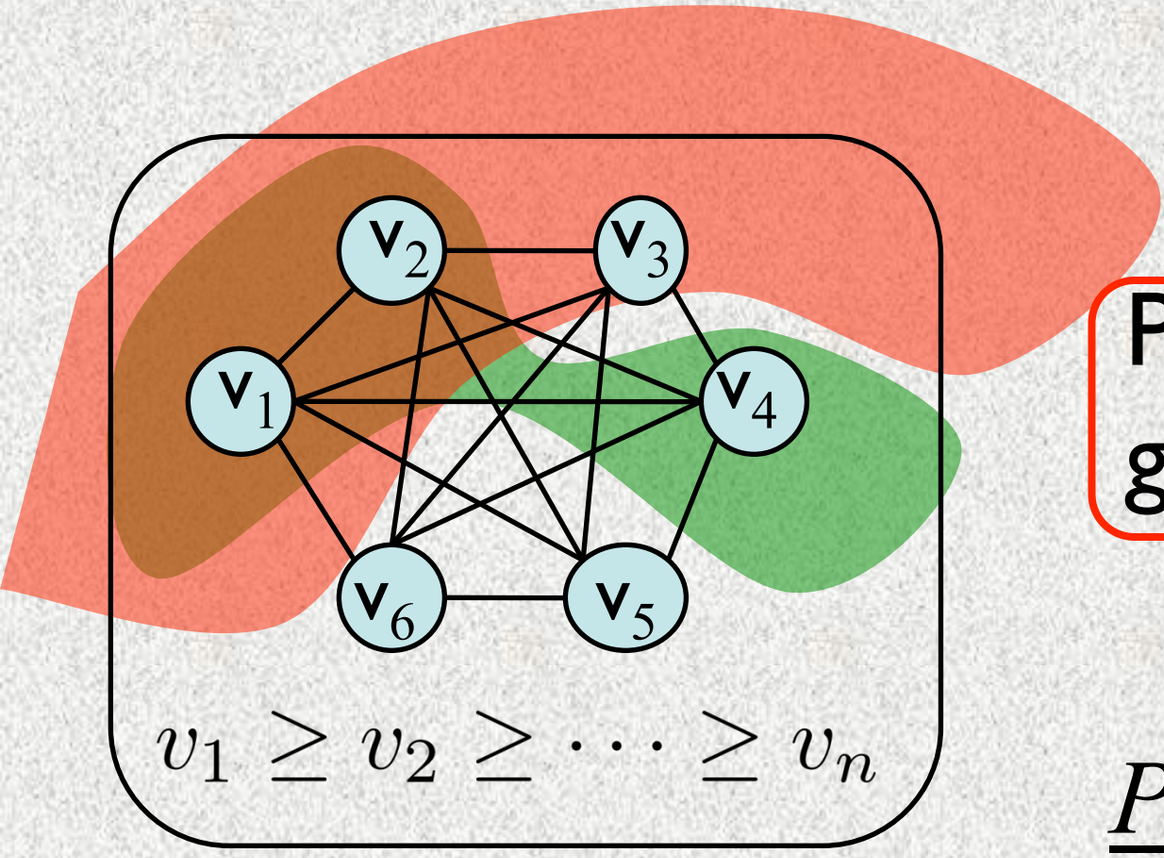
Social opt (SO)

Winning group (W)

Property 1: Intersection of SO and W is non-empty

Proof: Only the common (intersecting) members of W and SO can prevent SO from forming and winning.

SPoA for MAGNET game



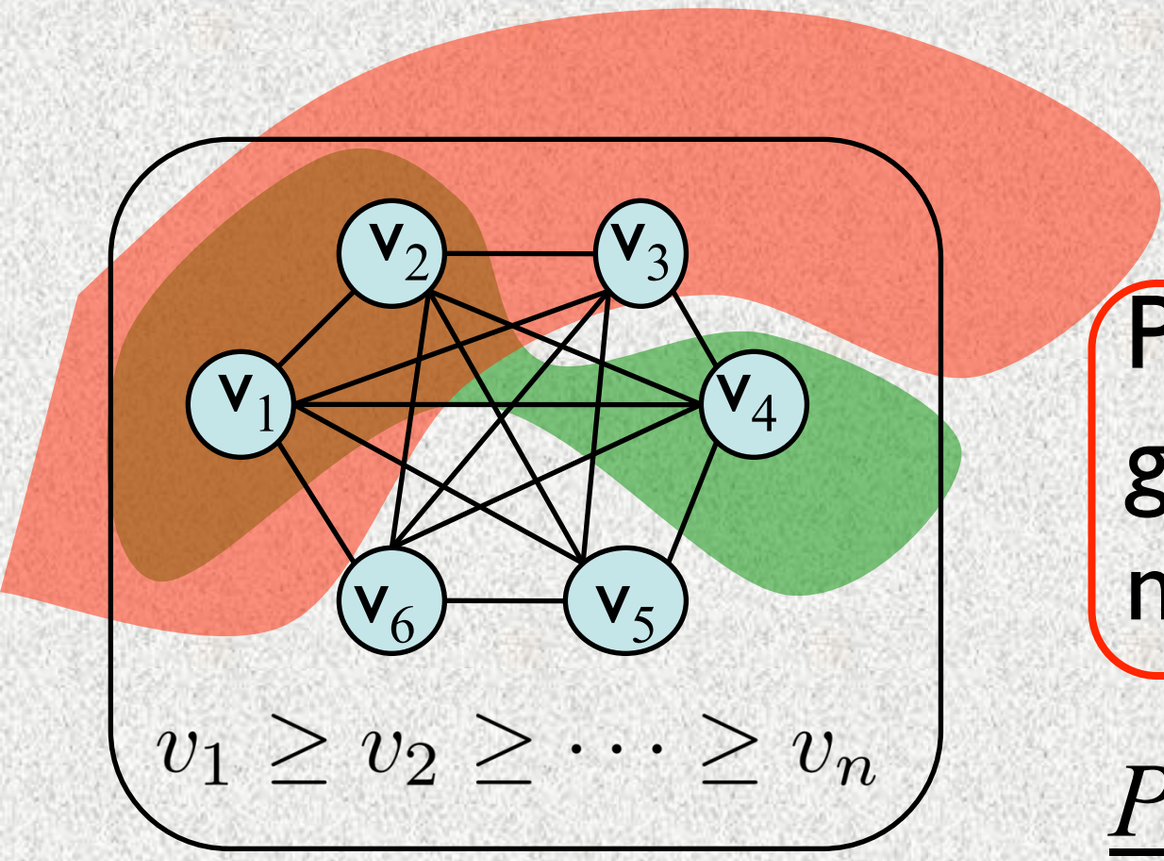
Social opt (SO)

Winning group (W)

Property 2: Size of W is not greater than Size of SO

Proof: The common (intersecting) members of W will only form W (and not SO) if their utility improves.

SPoA for MAGNET game



Social opt (SO)

Winning group (W)

Property 3: Average of W is greater than Average of (SO minus W)

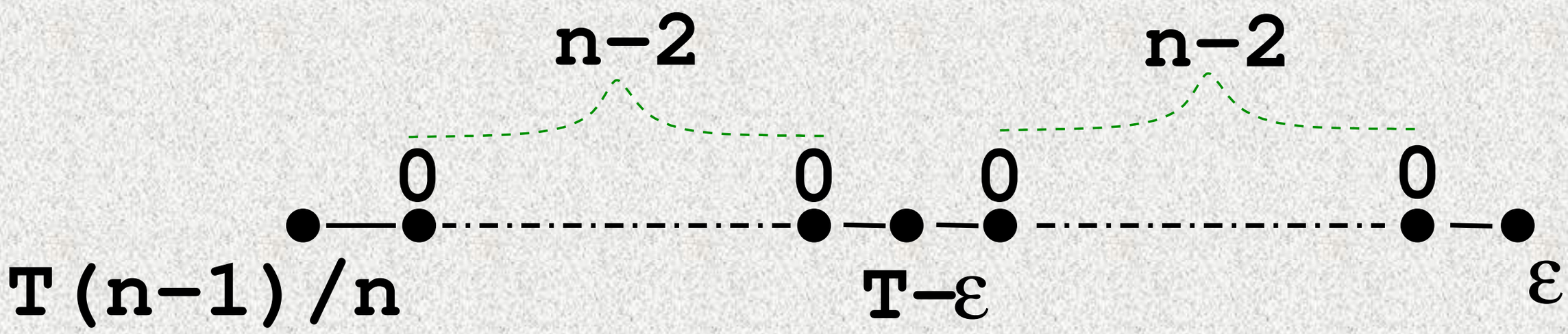
Proof:

Otherwise by appeal process, members of (SO minus W) will join.

SPoA for MAGNET game

✧SPOA of the MAGNET game is at most 2

✧SPOA of the MAGNET game is arbitrarily close to 2



SPoA for MAGNET game

SPOA of the MAGNET game is at most 2

Proof: Let k = Size(Social-Opt), SOW = SO, Z = W_r

$$SPOA = \frac{avg(SOW)}{avg(Z)} = \frac{sum(SOW \cap Z)/k + sum(SOW \setminus Z)/k}{avg(Z)}$$

1) $sum(SOW \cap Z)/k \leq avg(Z)$

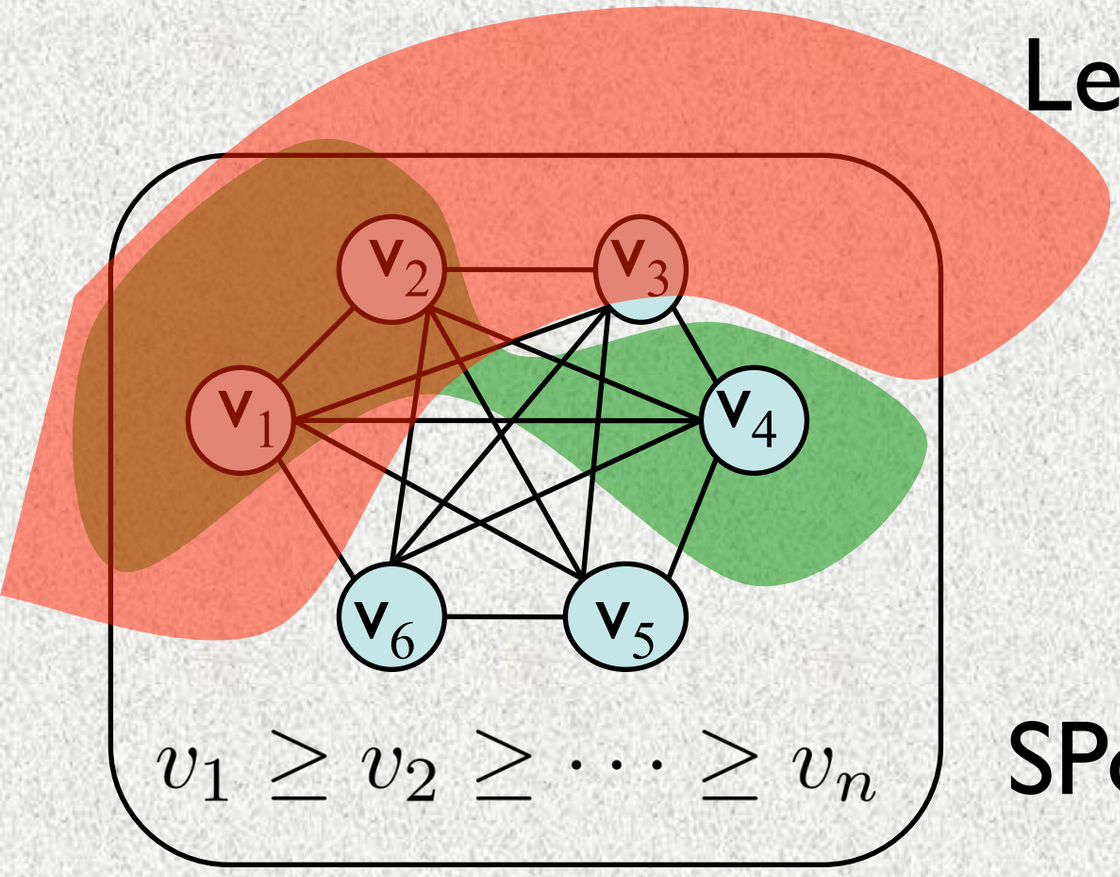
$$\frac{sum(SOW \cap Z)}{k} \leq \frac{sum(Z)}{k} \leq \frac{sum(Z)}{|Z|} = avg(Z)$$

2) $sum(SOW \setminus Z)/k \leq avg(Z)$

$$\frac{sum(SOW \setminus Z)}{k} \leq \frac{sum(SOW \setminus Z)}{|SOW \setminus Z|} = avg(SOW \setminus Z) \leq avg(Z)$$

Therefore, $$SPOA \leq \frac{avg(Z) + avg(Z)}{avg(Z)} = 2$$

SPoA for MAGNET game



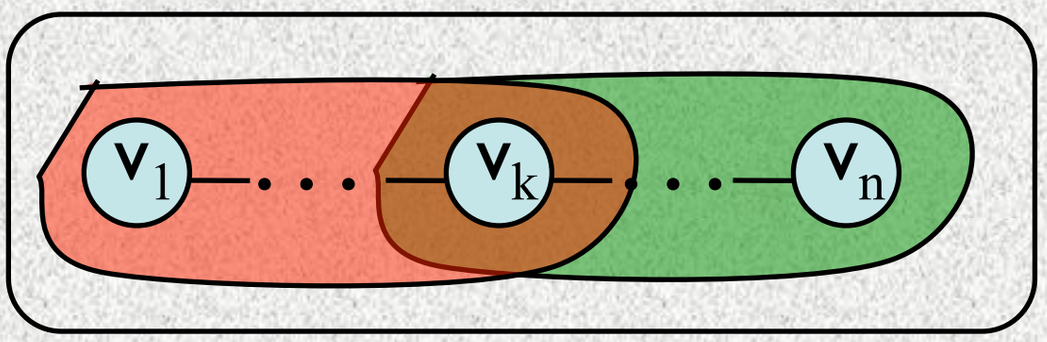
Let T = threshold

k = Size(Social-Opt)

Social opt

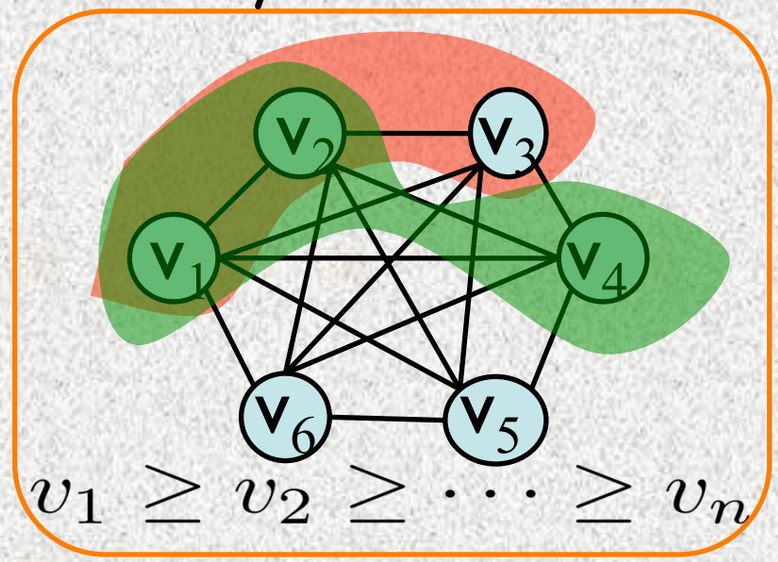
Winning group

$\text{SPoA}_{\text{clique}} = 1 + 1/k$



$\text{SPoA}_{\textit{line}} = 1 + (k-1)/k$

Proof Outline for SPOA$_{complete}$ = $1 + 1/k$



- Arrange in sorted order

Then, Social Opt = $\{v_1, \ldots, v_k\}$

e.g. k = 3; $v_1 + v_2 + v_3 > T$

- Let W = Winner group. We know Size(W) = k

e.g. let W = $\{v_1, v_2, v_4\}$

- Thus, $v_4 \leq v_3 \leq Sum(W)/k$ (else $W_r \neq W$)

$$SPOA = \frac{Sum(OPT)/k}{Sum(W)/k} = \frac{v_1 + v_2}{Sum(W)} + \frac{v_3}{Sum(W)} \leq 1 + \frac{1}{k}$$

Main Results

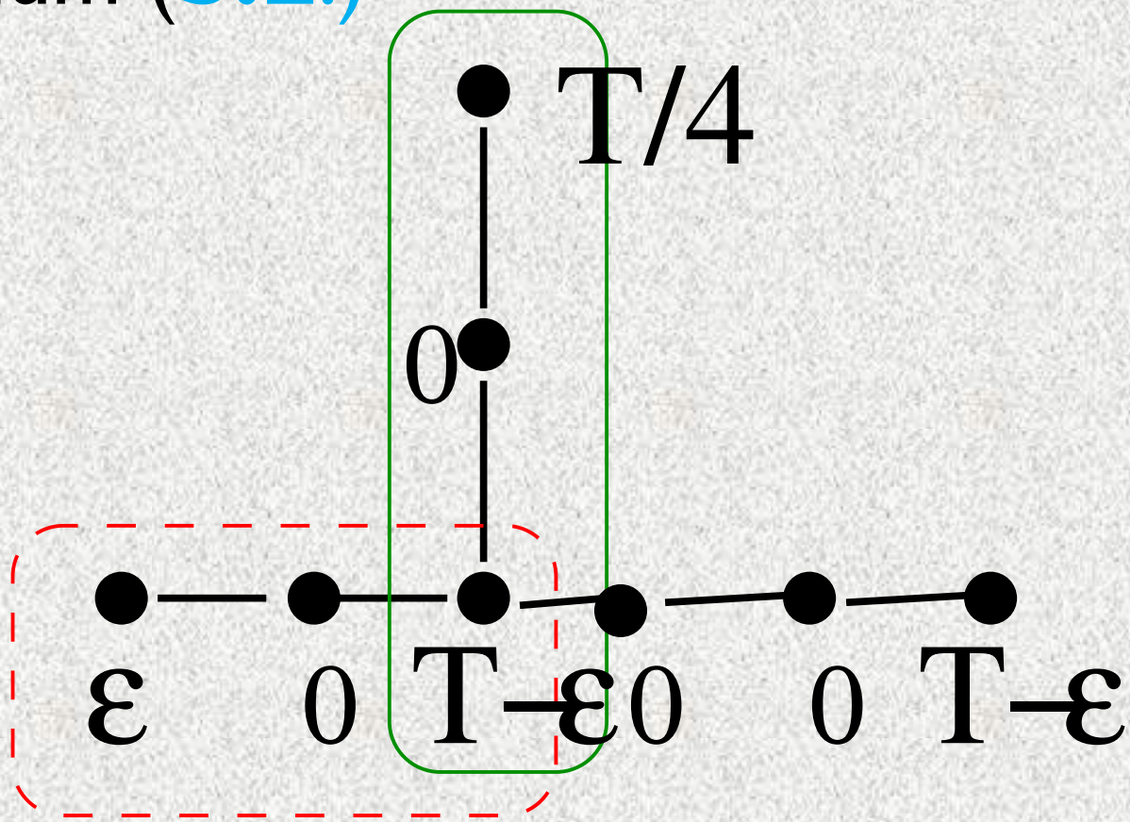
- For the *GoldRush game*, there is no strong equilibria; PoA is n/2 (for n players)
- For the *CCC game*, SPoA can be up to 2 (Complete graphs) or 3 (arbitrary graphs)
- For the *MAGNET CCC game*:
  - ✧ SPoA is at most 2
  - ✧ For k = Size(OPT); SPoA over a complete graph is $1 + 1/k$
  - ✧ SPOA over a line graph is $1 + (k-1)/k$

Additional Results: SPOS

Strong Price of Anarchy: SPoS

$$\text{SPoS} = \frac{\text{Social Optimum}}{\text{Best Strong Equilibrium (S.E.)}}$$

- SPOS in MAGNET can be greater than **1**



SPOS = 6/5

SPOA = 3/2

Additional Results: Strong Subgame Perfect Equilibria (SSPE)

For SSPE, The SPOA is the same as that for SE for the MAGNET game

- Every SSPE is also a SE => SSPE can only decrease SPOA.
- If W is the SE MAGNET winner, there is a game for which W is the SSPE winner.

Extensions: Coverage

- Coverage: researchers have a set of skills and the feasible group should have a required skill set.

Extensions: Synergy

- A function f: Group -> Value
- Analyse our games for such f.
- Superadditivity?: f(x + y) >= f(x) + f(y)

Extensions: Reducing the number of rounds of the MAGNET game

- The 2-round MAGNET game?

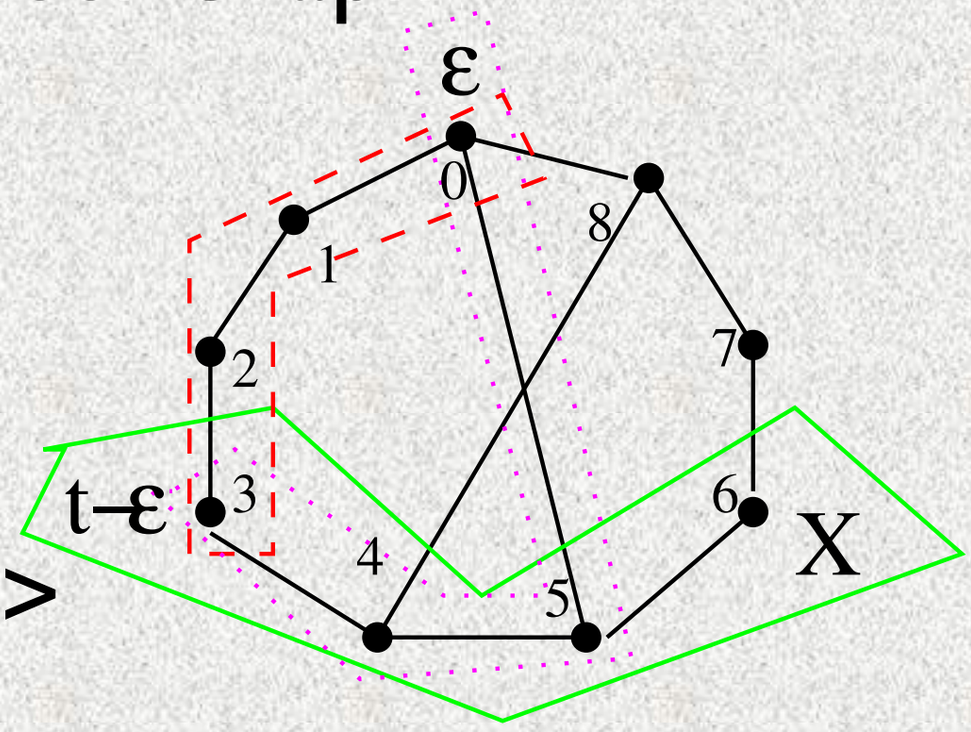
Extensions: Bounded Rationality

- Relaxing the assumption that the players/ researchers are fully rational.

Extensions: Topology dependence

- How exactly does the SPOA vary with topological properties (diameter/ connectivity)?
- Ideas: 3-Non-Zero Games (3NZ Games) and the Cartwheel Graph

SPOA = 1.66 ->



Future Work

- Is MAGNET the best mechanism given the assumptions?
- Reducing the number of rounds of the MAGNET game
- Deriving relationships with topological properties
- Complexity of the Equilibria: In MAGNET, Finding OPT and any Equilibria are NP-Hard. Can we get better mechanisms?
- Varying Utility: Negotiation between players?, Grant money as a function of consortum size?

Future Work

- Studying related 'natural' games (not designing mechanisms)
- *Dynamic* environment: Old Researchers retire, new researchers are born!
- Distributed computation: Can nodes compute equilibria with limited local knowledge?

We wish everybody a successful gold rush in their grant proposals.

-and please keep us in mind as members in your groups applying for EU (or NSF) grants…