RW2: Liquidity in Credit Networks

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Credit Network

- Decentralized payment infrastructure introduced by [DeFigueiredo, Barr, 2005] and [Ghosh et. al., 2007]
- Do not need banks, common currency
- Models trust in networked interactions
- A robust "reputation system" for transaction oriented social networks

Barter and Currency

- Barter: If I need a goat from you, I had better have the blanket that you are looking for. Low liquidity.
- Centralized banks: Issue currencies, which are essentially IOUs from the bank. Very high liquidity; allows strangers to trade freely.
- Credit Networks: Bilateral exchange of IOUs among friends.

Illustration: Credit Networks

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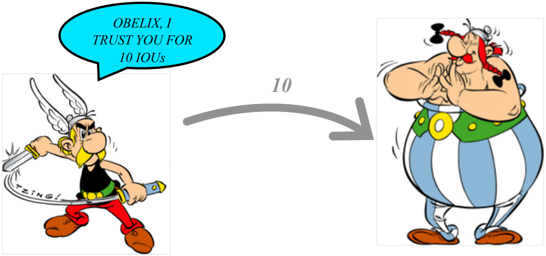

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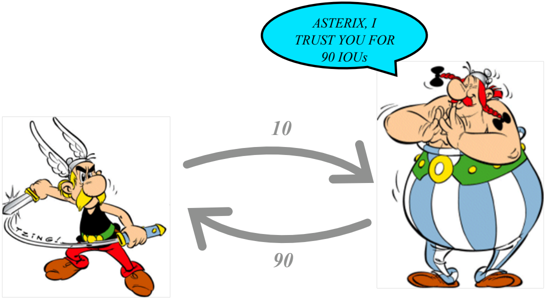

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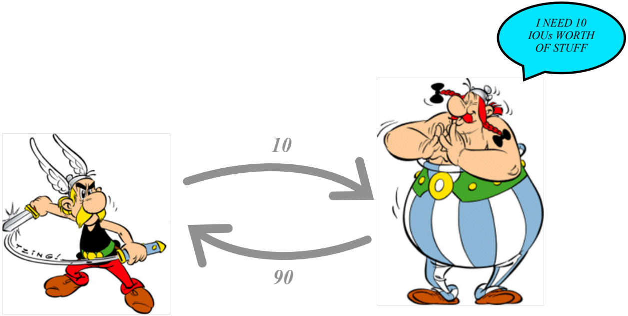

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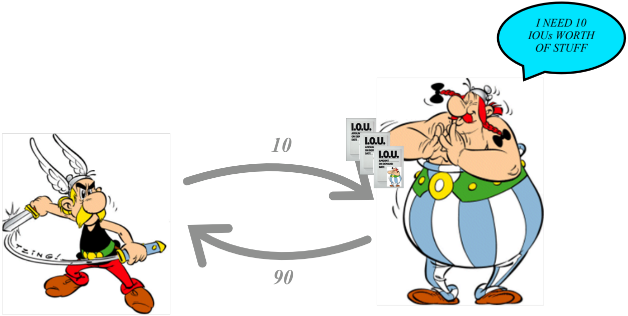

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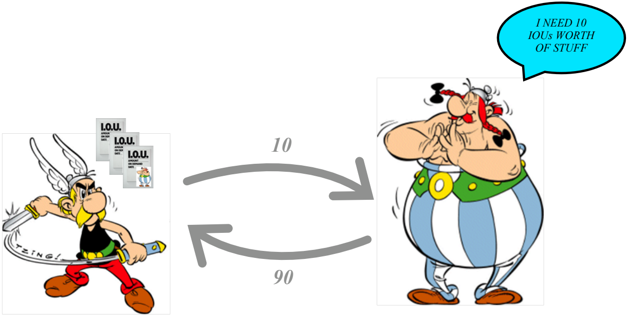
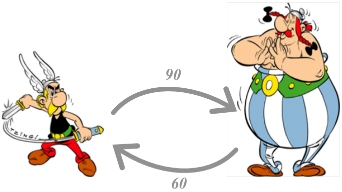


Illustration: Credit Networks

New Trust Values…

Illustration: Credit Networks

Interaction at a Distance



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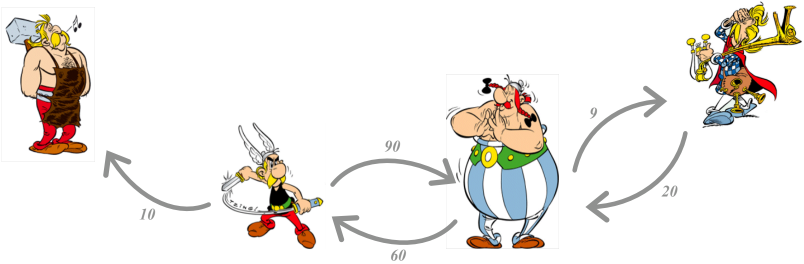

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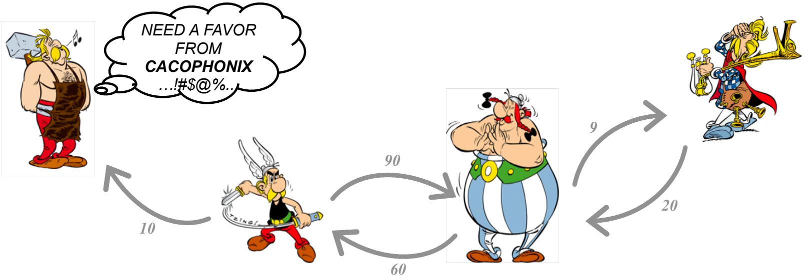

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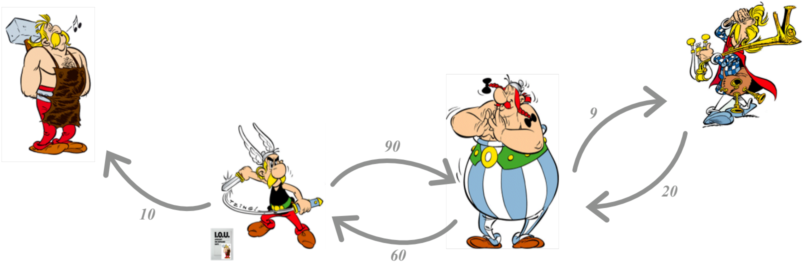

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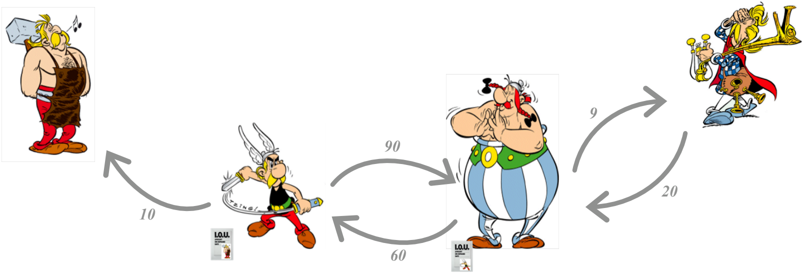

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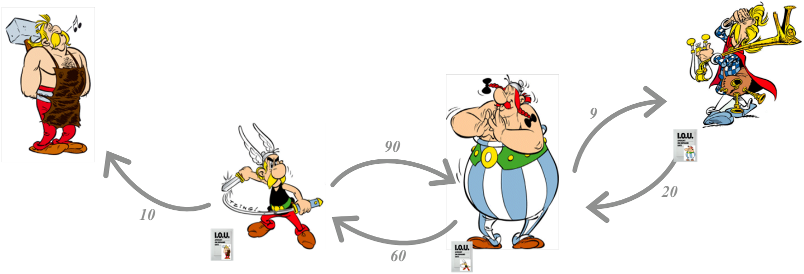

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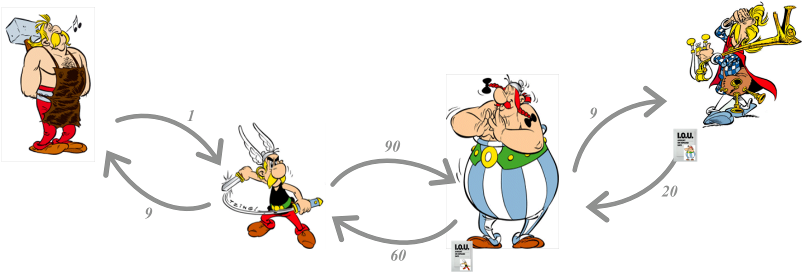

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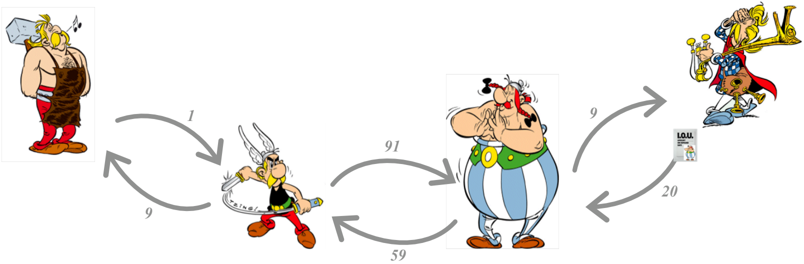

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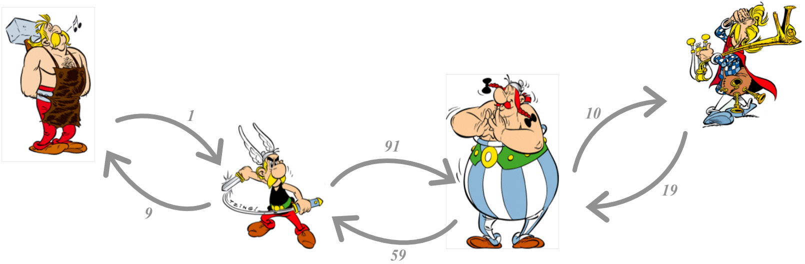


What is a Credit Network?

- Graph $G(V, E)$ represents a network (social network, p2p network, etc.)
- **Nodes:** (non-rational) agents/players; print their own currency
- **Edges:** credit limits $c_{uv} > 0$ extended by nodes to each other[1]
- Payments made by passing IOUs along a chain of trust. Same as augmentation of *single-commodity* flow along the chain
- Credit gets replenished when payments are made in the other direction

Robustness: Every node is vulnerable to default only from its own neighbors, and only for the amount it directly trusts them for.

[1] assume all currency exchange ratios to be unity

Research Questions

- Liquidity: Can credit networks sustain transactions for a long time, or does every node quickly get isolated?
- Network Formation: How do rational agents decide how much trust to assign to each other?

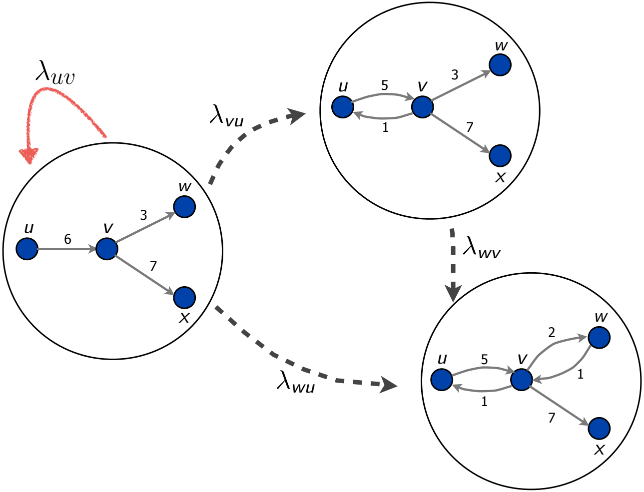
Liquidity Model

- Edges have integer capacity $c > 0$ (summing up both directions)
- Transaction rate matrix $\Lambda = \{\lambda_{uv} : u, v \in V, \lambda_{uu} = 0\}$
- Repeated transactions; at each time step choose (s, t) with prob. λ_{st}
- Try to route a unit payment from t to s via the shortest feasible path; **update edge capacities** along the path
- Transaction fails if no path exists

Liquidity Model

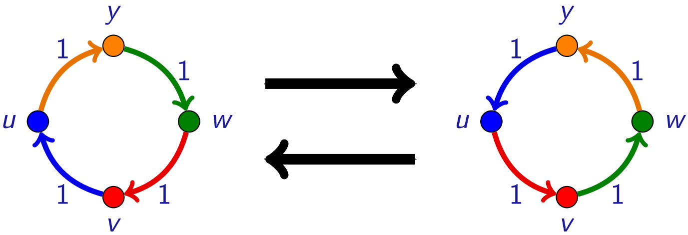
The Random Walk

Failure rate = Stationary probability of making a transition to the same state



Analysis

Cycle-reachability



Definition

Let $\mathcal{S}$ and $\mathcal{S}'$ be two states of the network. We say that $\mathcal{S}'$ is **cycle-reachable** *from $\mathcal{S}$ if the network can be transformed from state $\mathcal{S}$ to state $\mathcal{S}'$ by routing a sequence of payments along feasible cycles (i.e. from a node to itself along a feasible path).*

Analysis

Steady-State

Cycle-reachability partitions all possible states of the credit network into equivalence classes.

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If the transaction rates are symmetric, then the network has a uniform steady-state distribution over all reachable equivalence classes.

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Consequence: Yields a complete characterization of success probabilities in trees, cycles, or complete graphs; estimate for Erdös-Rényi graphs

Analysis

Example: Two node network

Assume capacity c. Then we have $c + 1$ states; each in a different equivalence class.

Success probability for a transaction is $c/(c + 1)$.

Analysis

Example: Tree networks

No cycles. Hence, all states are equally likely.

Let $c_1, c_2, \ldots, c_L$ be the capacities along the path from s to t in the tree. Then, success probability is

$$\prod_{i=1}^{L} c_i/(c_i + 1).$$

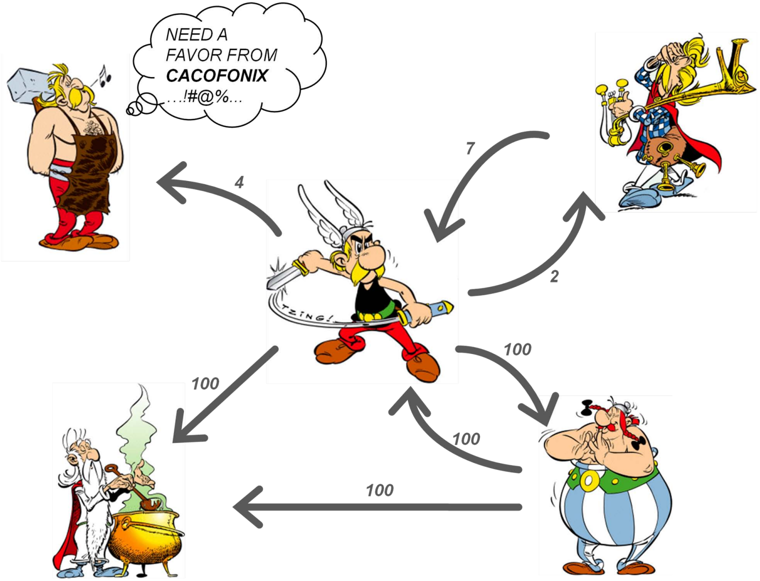
Analysis

Example: Bankruptcy probability in general graphs

Assume capacity $c = 1$ on each edge, and the Markov chain is ergodic. Let d_v denote the degree of node v. Then the stationary probability that v is bankrupt is at most $1/(1 + d_v)$.

Analysis

Centralized Payment Infrastructure



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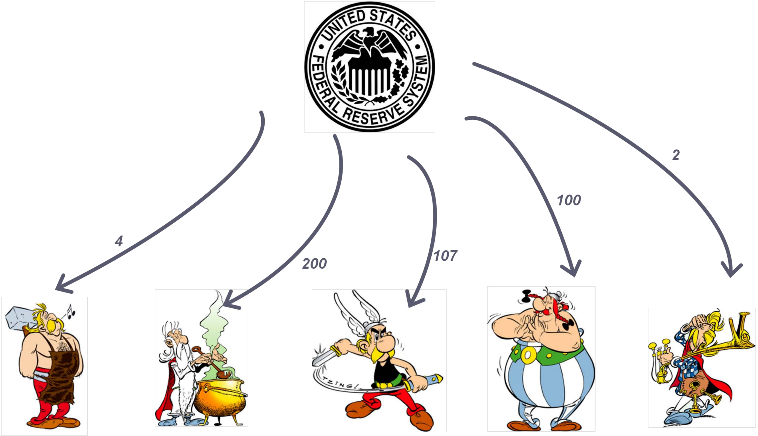

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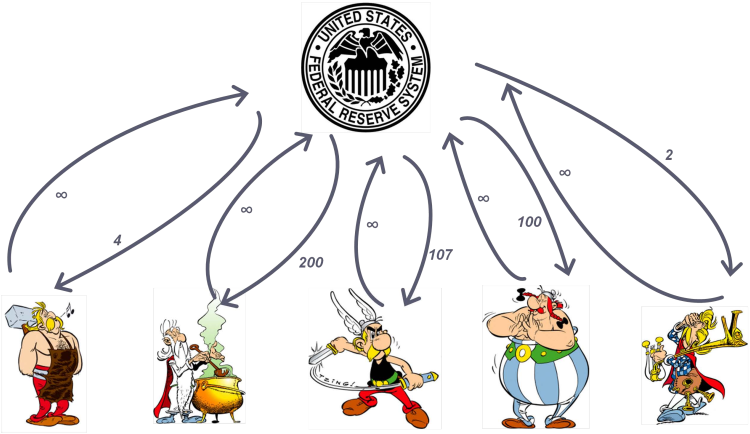

Analysis

Centralized Payment Infrastructure

Convert Credit Network → Centralized Model

$$\forall u, c_{ru} = \sum_{v} c_{vu}$$

Analysis

Centralized Payment Infrastructure

Convert Credit Network $\rightarrow$ Centralized Model

$$\forall u, c_{ru} = \sum_{v} c_{vu}$$

$\Longrightarrow$ Total credit in the system is conserved during conversion

Analysis

Centralized Payment Infrastructure

Convert Credit Network → Centralized Model

$$\forall u, c_{ru} = \sum_{v} c_{vu}$$

$\implies$ Total credit in the system is conserved during conversion

Slight variant of the liquidity analysis gives steady state distribution and success probabilities.

Liquidity Comparison

Dandekar, Goel, Govindan, Post; 2010

Bankruptcy probability

Graph class	Credit Network	Centralized System
General graphs	$\leq 1/(d_v + 1)$	$\approx 1/(d_{AVG} + 1)$

Transaction failure probability

Graph class	Credit Network	Centralized System
Star-network	$\Theta(1/c)$	$\Theta(1/c)$
Complete Graph	$\Theta(1/nc)$	$\Theta(1/nc)$
$G_c(n, p)$	$\Theta(1/npc)$ (simulation/estimate)	$\Theta(1/npc)$

Summary: Many credit networks have liquidity which is almost the same as that in centralized currency systems.

Random Forests

An Interesting Connection

- $G = (V, E)$, a multi-graph,
- RF-connectivity between two vertices u and v = Pr(u is connected to v in a uniformly chosen random forest of G).

Prop: Liquidity in a Credit Network = Average RF-connectivity in the underlying graph (via [Kleitman and Winston, 1981])

Liquidity in Expander Graphs

Goel, Khanna, Raghavendra, Zhang; 2015

Def: Expansion of a graph is

$$h(G) = \min_{S \subseteq V :\, 0 \leq |S| \leq |V|/2} \frac{|E(S, \bar{S})|}{|S|}$$

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For graphs with expansion $h(G)$,

Thm (Main): Average RF-connectivity over any two vertices $\geq 1 - \frac{2}{h(G)}$.

Thm: Average RF-connectivity between one vertex and all other vertices $\geq 1 - \frac{\log n + 2}{h(G) + 1}$.

Corollaries

Corollaries: In a uniformly random forest,

- Expected size of largest component $\geq n - \dfrac{2n}{h(G)}$.
- Expected number of components $\leq 1 + \dfrac{2n}{h(G)}$.
- $\Pr(\text{largest component} \leq \dfrac{n}{2}) \leq \dfrac{2}{h(G)}$.

RF-connectivity on Expanding Subgraphs

Thm: Let S be any subset of vertices and G_S be the induced subgraph. Then $\Phi_S(G) \geq 1 - \frac{2}{h(G_S)}$.

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The Monotonicity Cojecture: RF-connectivity can not decrease if we add a new edge in the graph.

Equivalent to Negative Correlation (known for random spanning trees).

Open Problems

- The Monotonicity conjecture
- Approximately sampling a random forest from a graph
- Rationality: how do nodes initialize and update trust values (in general settings)?

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