Prediction Markets

Boi Faltings

AI Laboratory, EPFL
boi.faltings@epfl.ch
http://liawww.epfl.ch/

2016

Prediction Aggregation

Multiple agents have a probability estimate for a phenomenon.
How do we aggregate this information?

- weight according to confidence.
- confidence should be elicited truthfully.

Prediction Markets

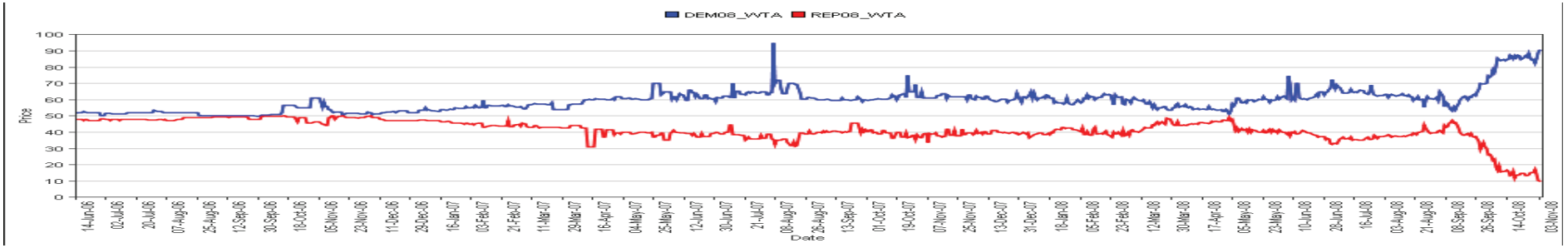
Model on a financial market:

- Market = trade securities $s(x_i)$ for predictions x_i that pay \$1 if $g = x_i$ and \$0 otherwise.
- every security has a market price $c(x_i)$.
- agents "invest" their money in a prediction by buying $s(x_i)$. $c(x_i) < Pr(x_i) \Rightarrow buy(s(x_i))$, otherwise sell.
- competitive equilibrium = $c(x_i)$ is a consensus probability estimate for $Pr(g = x_i)$.
- bigger investment $\Leftrightarrow$ bigger influence, but also risk.

$\Rightarrow$ agents invest their budget where they are the most confident.

Example: Iowa Electronic Market

- US 2008 presidential election (Iowa Electronic Market):



- In all recent elections, this market was more accurate than opinion polls!
- Other applications: predict project completion, market prospects, etc.
- Mainly used internally in large organizations.

Liquidity and Market Makers

- Participants in a market must have someone to trade with.
- Market-maker: agent that is committed to trade at any time and with any counterparty at some price.
- How to construct such a market maker for prediction markets?

Scoring rules revisited

- Suppose we predict which of k different values $r_1, .., r_k$ will be taken by a variable r. A participant believes the distribution to be p.
- Scoring rule mechanism: the participant reports an estimated probability distribution $\hat{p}$ over the outcomes, and is rewarded $s_i(\hat{p})$, where r_i is the true outcome.
- Reward $s_i(\hat{p})$ should be:
  - maximal when $\hat{p} = p$ (to ensure truthfulness)
  - ≥ 0 in expectation, i.e. $\sum_i p_i s_i(p) \geq 0$ (to ensure participation)

$\Rightarrow$ proper scoring rule.

Examples of proper scoring rules

- Quadratic scoring rule:

$$s_i = a_i + b\left(2\hat{p}_i - \sum_k \hat{p}_i^2\right)$$

- Logarithmic scoring rule:

$$s_i = a_i + b_i \ln \hat{p}_i$$

- Logarithmic scoring rule the only one that generalizes to dependent events.

Automated market makers

- Consider a simple market with one security that pays 1 if even r occurs.
- Market maker buys/sells securities at a current price.
- Let $p(n)$ be the price for one security given that n securities have been bought, and $c(n)$ be the cumulative cost paid by all participants.
- Q: What price function makes the price show the estimated probability?
- A: All participants together should be paid for the final result of the decision market according to a truthful scoring rule.

Market makers with logarithmic scoring rules

- Assume participant believes that true probability of outcome 1 should be $p' > p$, and buys/sells m securities until $p(n+m) = p'$.

$\Rightarrow$ he should make a profit of $s_1(p') - s_1(p)$ if the outcome is indeed 1:

$$\begin{aligned} m(1-p) &= s_1(p(n+m)) - s_1(p(n)) \\ (1-p) &= \frac{ds(p(n))}{dn} = \frac{ds}{dp}\frac{dp}{dn} \end{aligned}$$

- For logarithmic scoring rule $s = b \ln p$, solved by:

$$p(n) = \frac{e^{n/b}}{e^{n/b} + 1}$$

Issues with Logarithmic Market Makers

- Price can never reach 1: what to do in case of certain events?
- As price approaches 1, making gains requires buying huge numbers of securities, thus taking huge risks!

$\Rightarrow$ most suitable for problems with quite uncertain outcomes.

Prediction markets in practice

- Prediction markets are widely used.
- Several companies market software for prediction markets: Microsoft, Inkling markets, etc.
- Biggest successes in predicting events in companies.
- But not clear if market mechanism is better than opinion polls!

K.J. Arrow et al.: "The promise of prediction markets." SCIENCE 320.5878, p. 877, 2008