# Novel algorithms for distributed spectrum sensing

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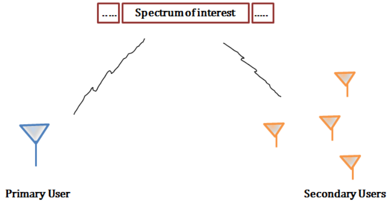
# Outline

- Introduction - CR and spectrum sensing
- Non parametric Sequential Hypothesis Testing algorithms
- Performance Comparison
- Asymptotic properties
- Robust tests
- Distributed set up
- Putting together what we learnt
- Performance analysis of distributed algorithm
- Asymptotics of distributed algorithm
- Conclusion

# Introduction: Cognitive Radio




- CR: perform radio environment analysis, identify spectral holes and operate in those holes.
- Licensed (primary) and Unlicensed (secondary) users
- Spectrum Sensing
  - Identify spectral holes and quickly detect the onset of primary transmission
  - Primary techniques: Matched filter, Cyclostationary detector and Energy detector
  - Challenges: Shadowing, Fading and low SNR ($\sim -20dB$), sensing frequency and duration

# Model for Cognitive Radio

## Non parametric sequential hypothesis testing problem

- i.i.d. observations $X_i, i = 1, 2, \ldots$

$$\begin{aligned} H_0 &: X_i = N_i && \sim P_0(\text{ p.d.f. } f_0) \\ H_1 &: X_i = H_i S_i + N_i && \sim P_1(\text{ p.d.f. } f_1) \end{aligned}$$

- Take $N$ observations and use decision rule $\delta_N$ which

$$\min_{N,\delta_N} E[N|H_0] \text{ and } \min_{N,\delta_N} E[N|H_1],$$

$$\text{subject to } P_{FA} \leq \alpha \text{ and } P_{MD} \leq \beta$$

# Issues in Spectrum sensing

- -20 dB SNR, shadowing, fading $\Rightarrow$ Distributed detection required.
- Fast detection $\Rightarrow$ Sequential detection preferred.
- Transmit power, channel gains, modulation, coding schemes etc. of PU transmissions are not known (SNR uncertainty)
  $\Rightarrow$ (Generalised) Energy detection optimal.
- $H_i S_i$ distribution not known
  $\Rightarrow$ Non parametric/Semi parametric and composite hypothesis.
- Fast fading - Rayleigh, Rician, Nakagami.
  Slow fading - Log normal.
- Time varying electromagnetic interference $\Rightarrow$ Distribution of SINR $N_i$ may not be known and noise power time varying.
- Outliers present $\Rightarrow$ Robust tests desired.
- Hence, Non parametric, distributed, robust, sequential tests preferred.

# Existing methods - a discussion

- $P_0$ and $P_1$ fully known - SPRT optimal for discrete and continuous alphabet, For $0 < \alpha, \beta < 1$

$$N \triangleq \inf\{n : \tilde{W}_n = \sum_{k=1}^{n} \log \frac{P_1(X_k)}{P_0(X_k)} \notin (\log \beta, -\log \alpha)\},$$

$$\delta_N = H_1 \text{ if } \tilde{W}_N \geq -\log \alpha;\ H_0 \text{ if } \tilde{W}_N \leq \log \beta$$

- $P_0$ is known, $P_1$ is not known - our algorithm (KTSLRT).
  - Discrete alphabet case: $L_n(X_1^n)$, codelength of a universal lossless source code for $X_1^n$. We replace $\tilde{W}_n$ by

$$\widehat{W}_n = -L_n(X_1^n) - \log P_0(X_1^n) - n\frac{\lambda}{2},\ \lambda > 0.$$

  - $\lambda$ chosen to make it Order 1 asymptotically optimal

# Existing methods - a discussion

- Continuous alphabet case:
  - Quantize $X_k$ to $X_k^\Delta = [X_k/\Delta]\Delta$ (Uniform scalar quantization)
  - Range of quantization according to $f_0$'s tail probabilities less than a small specific value.
  - Use Universal lossless coding on $X_1^\Delta, X_2^\Delta, \ldots, X_n^\Delta$
- Performs better than the asymptotically optimal Hoeffding test for finite alphabet sources, Kolmogorov-Smirnoff test and some other non parametric tests.

# Our new algorithm - entropy test

## Discussion of the test

- $P_0$ is known, $P_1$ is not known.
  - Replace $\widehat{W_n}$ by

    $$W_n = W_{n-1} + [-\log P_0(X_n) - H(P_0) - \frac{\lambda}{2}],\ \lambda > 0.$$

    where $H(P_0)$ is entropy (differential entropy for continuous case) of $P_0$.
  - Average drift under $H_1$: $D(P_1||P_0) + H(P_1) - H(P_0) - \lambda/2$
    Thus we take under $H_1$, $\{P_1 : D(P_1||P_0) \geq \lambda, H(P_1) \geq H(P_0)\}$
  - Average drift under $H_0$: $-\lambda/2$.
  - Does not require quantization for continuous case.
  - Does not require a Universal source encoder.

# Other non parametric tests

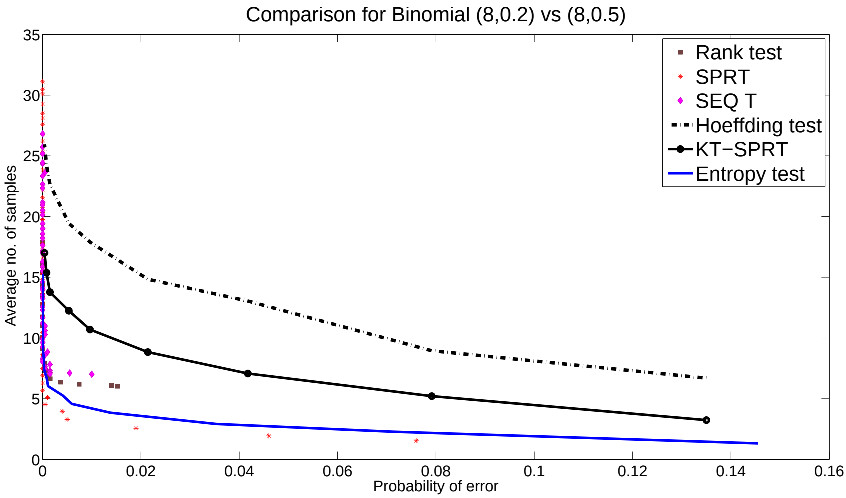
- Rank Test
  i. Let $Y_i = X_i - \frac{\mu_0+\mu_1}{2}$, where $X_i$s are the observations.
  ii. Calculate $R_i$, the rank of $Y_i$ in $Y_1, ..., Y_n$ when arranged in ascending order of their absolute values.
  iii. Test statistic is given by, $W = \sum_{i=1}^{n} Y_i \frac{R_i}{n+1}$.
  iv. Non parametric, for symmetric distributions.
  v. Use its sequential version.

# Other non parametric tests

- Sequential t test
  i. We extend the usual t test to make it a two sided test.
  ii. Test statistic is given by, $T_n = n\frac{\bar{X}_n - \frac{\mu_0+\mu_1}{2}}{s_n}$.
- Random walk
  i. Test statistic is given by, $T_n = \sum_{i=1}^{n}(\bar{X}_n - \frac{\mu_0+\mu_1}{2})$.
  ii. The test statistic is iteratively computable.

# Performance Comparison

Comparison with different tests: $P_0 \sim Bin(8, 0.2)$, $P_1 \sim Bin(8, 0.5)$

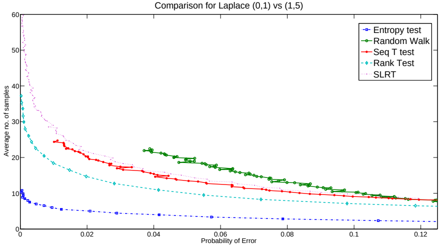


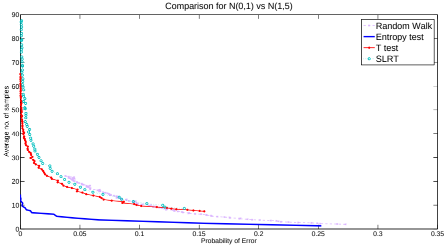
# Performance Comparison

## Laplace and Gaussian distributions example

$f_1 \sim Laplace(1,5)$
$f_0 \sim Laplace(0,1)$

$f_1 \sim \mathcal{N}(1,5)$
$f_0 \sim \mathcal{N}(0,1)$

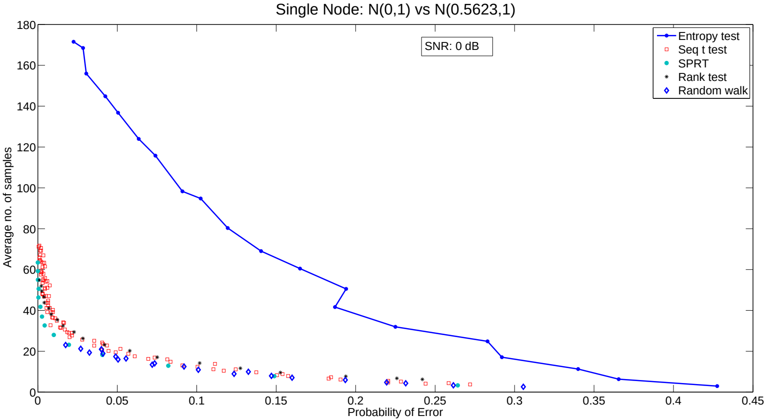



- Entropy test outperforms well known tests in all these cases.

# Performance Comparison

## Case of no variance change

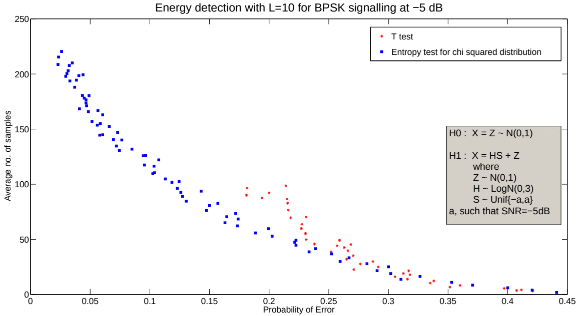



- Entropy test is not good when only mean difference tested.
- Rank test, t test and random walk perform much better.

# Performance Comparison

## Energy detection

- Energy samples are computed as: $Y_j = \sum_{i=1}^{L} X_i^2$




- Entropy test performs best when there is difference in variances; naturally arises in energy detection.
- Energy detection (adding up $|X_i|^2$) is a special case of summing up $|X_i|^p$ to form test statistics. The traditional method ($p = 2$) is optimal for additive Gaussian noise.

# Asymptotic Properties of entropy test

**Theorem**

(1) $P_0(N < \infty) = 1$ and $P_1(N < \infty) = 1$.

(2) $\limsup_{t \to \infty} \frac{N_0(t)}{t} \leq \frac{1}{\lambda(1/2+c)}$ *a.s.* and in $L_1$ (under $H_0$)

(3) $\limsup_{t \to \infty} \frac{N_1(t)}{t} \leq \frac{2}{\lambda}$ *a.s.* and in $L_1$ (under $H_1$)

(4) $P_{FA} \triangleq P_0(\widehat{W}_N \geq t_1) \leq c_1 e^{-\Gamma^* t_1}$
where $\log E_0[e^{\Gamma^* Z_1}] = 0$;
$Z_k = -\log P_0(X_k) - H(P_0) - \lambda/2$.

(5) $P_{MD} \triangleq P_1(\widehat{W}_N \leq t_0) \leq c_0 e^{-\Gamma^* t_0}$
where $\log E_1[e^{\Gamma^* Z_1}] = 0$;

# Asymptotic Properties of t test

## Theorem

(1) $P_0(N < \infty) = 1$ and $P_1(N < \infty) = 1$.

(2) Under $H_i$, $\frac{N(t)}{t} \to \frac{2\sigma}{\mu_1 - \mu_0}$ a.s., for $i = 0, 1$. Also, if $E_i[|X_1|] < \infty$ for $i = 0, 1$, then the convergence holds in $L_1$.

(3) $P_{FA} \leq \alpha^s$ where $s$ is the solution of $E_0[e^{s(X - \frac{\mu_0 + \mu_1}{2})}] = 1$

(4) $P_{MD} \leq \beta^{s^*}$ where $s^*$ is the solution of $E_0[e^{-s^*(X - \frac{\mu_0 + \mu_1}{2})}] = 1$

# Asymptotic Properties of random walk

**Theorem**

(1) $P_0(N < \infty) = 1$ and $P_1(N < \infty) = 1$.

(2) Under $H_i$, $\frac{N(t)}{t} \to \frac{2}{\mu_1 - \mu_0}$ a.s., for $i = 0, 1$. Also, if $E_i[|X_1|] < \infty$ for $i = 0, 1$, then the convergence holds in $L_1$.

(3) $P_{FA} \leq \alpha^s$ where $s$ is the solution of $E_0[e^{s(X - \frac{\mu_0 + \mu_1}{2})}] = 1$

(4) $P_{MD} \leq \beta^{s^*}$ where $s^*$ is the solution of $E_0[e^{-s^*(X - \frac{\mu_0 + \mu_1}{2})}] = 1$

# Robust Tests

- Use $\alpha$-trimmed mean for t test and random walk.
- M test - A robust version of $t$ test insensitive to the tails of the distribution.

$$T_n = \frac{\sum_{i=1}^{n} \psi(X_i - \frac{(\mu_0+\mu_1)}{2})}{(\sum_{i=1}^{n} \psi^2(X_i - \overline{X}_n))^{\frac{1}{2}}}$$

$\psi : \mathcal{R} \mapsto \mathcal{R}$, non decreasing, continuous, odd and bounded function

Huber suggested,

$$\psi_0(z) = \begin{cases} k & ,\text{if } z > k, \\ z & ,\text{if } |z| \leq k, \\ -k & ,\text{if } z < -k. \end{cases}$$

- Applying M trimming to random walk gives

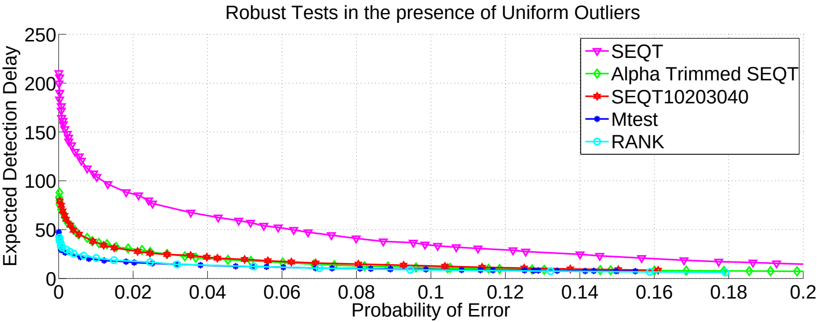
$$T_n = \sum_{i=1}^{n} \psi(X_i - \frac{(\mu_0 + \mu_1)}{2}).$$

This is iterative unlike $\alpha$-trimmed random walk/t test or M test.

# Robust tests

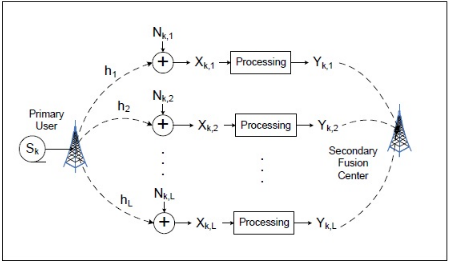
## Performance Comparison

- $F(x) = (1 - \epsilon)\Phi(x) + \epsilon H(x)$
  where $H(x) \sim$ Unif(7,8) or Unif(-8,-7) w.e.p.



# Distributed set up

## General algorithm




- Node $l$ receives $X_{k,l}$ and computes $W_{k,l}$.
- Node $l$ transmits $Y_{k,l}$ to the Fusion node.
- Fusion node receives $Y_k$ and computes $F_k$.
- Fusion node decides

$$H_0 \text{ if } F_k \leq \log \beta,$$
$$H_1 \text{ if } F_k \geq -\log \alpha;$$

otherwise continues.

$W_{k,l} = -\log P_0(X_1^n) - nH(P_0) - n\frac{\lambda}{2},\ \lambda > 0.$ (Entropy)

$$Y_{k,l} = b_1 \mathbb{I}\{W_{k,l} \geq -\log \alpha_l\} + b_0 \mathbb{I}\{W_{k,l} \leq \log \beta_l\}.$$

$$Y_k = \sum_{l=1}^{L} Y_{k,l} + Z_k.$$

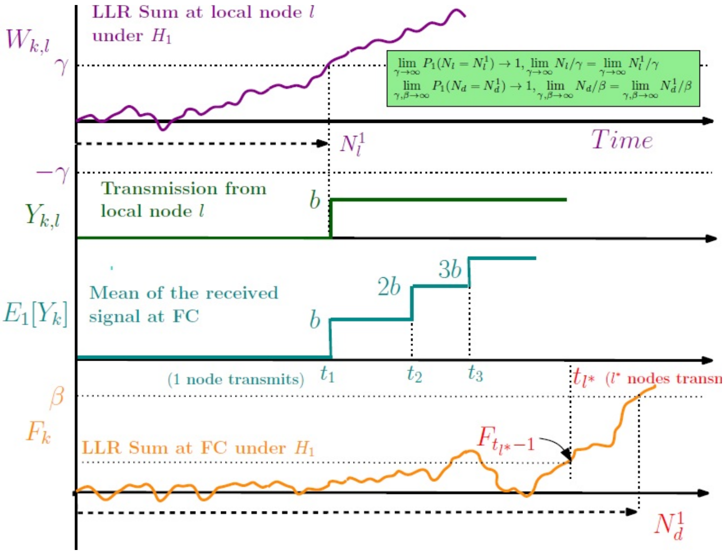
$F_k = F_{k-1} + \log \frac{g_{\mu_1}(Y_k)}{g_{\mu_0}(Y_k)},\ F_0 = 0.$ (SPRT)

# Comments

- Noise at FC usually not considered in literature.
  - With noise, usual fusion decision rules AND, OR, Majority etc. do not work.
  - Our algorithm implicitly does it.
  - We use physical layer fusion (no MAC transmission delays)
- Noise + interference distribution at FC may not be known
  $\Rightarrow$ Non parametric test desired.
- EMI at FC $\Rightarrow$ Outliers will be there $\Rightarrow$ Robustness desired.

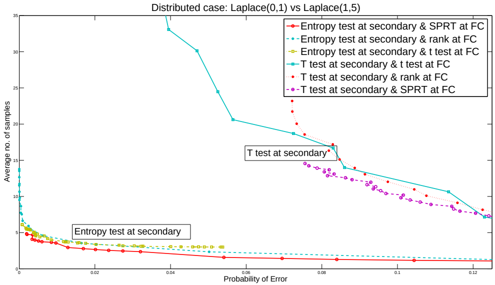
# Performance analysis of distributed algorithm

## Sample Path argument



# Different algorithms at FC and secondary nodes

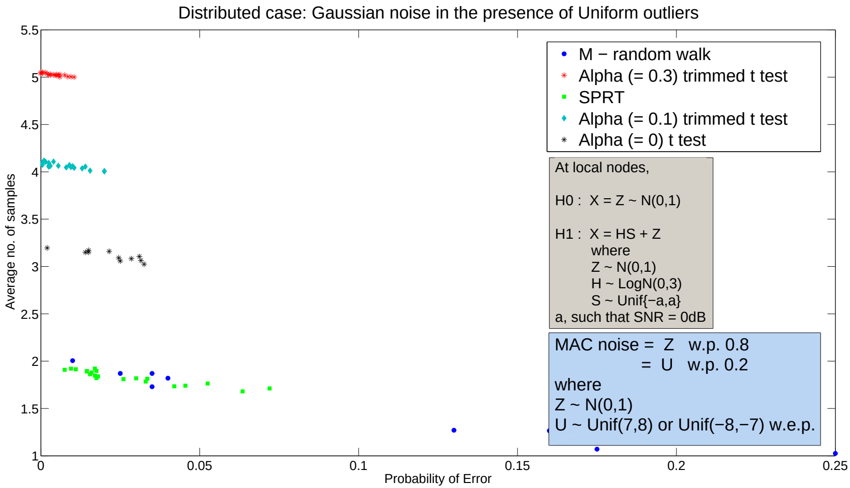
## for computing $W_{k,l}$ and $F_k$




- Entropy test performs best for energy detection (at secondary nodes).
- At the FC, only mean change, hence rank test/t test/random walk more appropriate.
- Rank test good only for symmetric distributions.
  Choose robust versions of random walk or t test.
- Conclusion: At local nodes, entropy test with energy detector.
  At FC, M-random walk/M test.

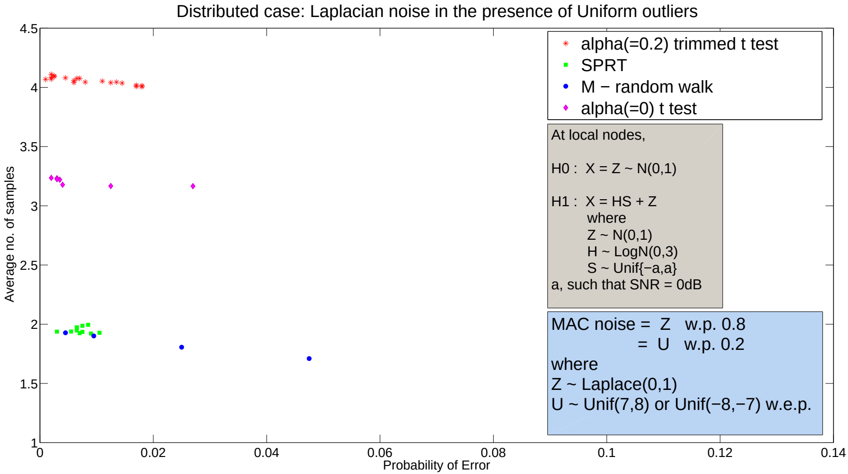
# Performance Comparison

## Gaussian noise (at FC) in the presence of Uniform outliers



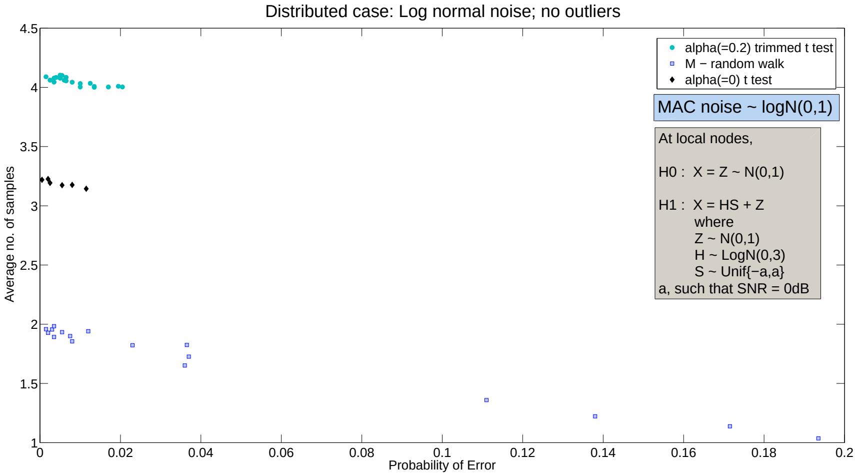
# Performance Comparison

## Laplacian noise (at FC) in the presence of Uniform outliers



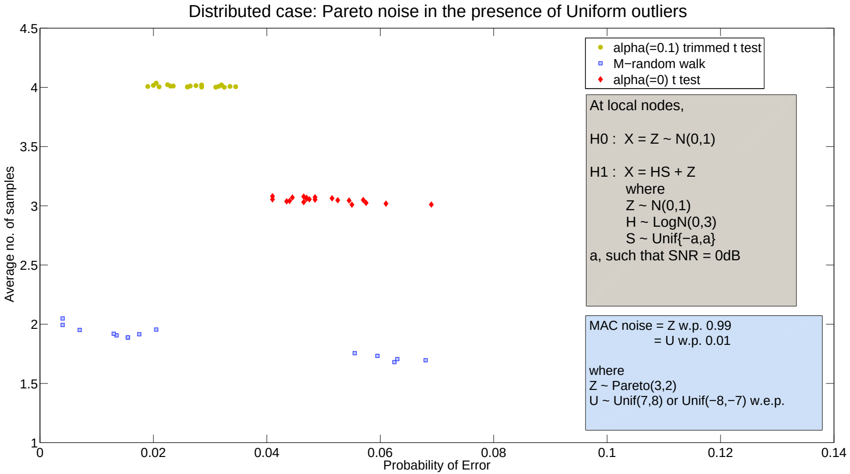
# More examples - heavy tailed without outliers

## Log normal noise (at FC) in the presence of Uniform outliers



# More examples - heavy tailed with 1% outliers

## Pareto noise (at FC) in the presence of Uniform outliers



# Asymptotics for distributed algorithm

$N$ = Time when FC makes decision
$P_{FA}$ = Probability of False alarm.
$P_{MD}$ = Probability of Missed Detection.
$\Gamma_{1,l} = \rho_l |logc|$, $\Gamma_{0,l} = -\Gamma_l |logc|$
$\beta_0 = -|logc|, \beta_1 = |logc|$

$D^i_{tot} = \sum_{i=1}^{L} E_1[X_{k,i} - \frac{\mu_0+\mu_1}{2}]$
$\zeta_1^* = Lb_1 + Z_k - \frac{\mu_0+\mu_1}{2}$
$\Delta_i = Lb_i - \frac{\mu_0+\mu_1}{2}$, i=0,1.
$\Gamma_l = \frac{E_0[X_{1,l} - \frac{\mu_0+\mu_1}{2}]}{D^0_{tot}}$
$\rho_l = \frac{E_1[X_{1,l} - \frac{\mu_0+\mu_1}{2}]}{D^1_{tot}}$

## Theorem

(1) $P_0(N < \infty) = 1$ and $P_1(N < \infty) = 1$ for any positive $\Gamma_{1,l}, \beta_1$ and any negative $\Gamma_{0,l}, \beta_0$.

(2) Under $H_i$, $\lim_{c \to 0} \frac{N}{|logc|} \le \frac{2}{L\lambda} + \frac{1}{\Delta_0}(1 + \frac{E_i[|\zeta_1^*|]}{D^i_{tot}})$, for i=0,1.

(3) If m.g.f. of $Z_1$ exists in a neighbourhood of 0, then
$\lim_{c \to 0} \frac{P_{FA}}{c} = 0$
$\lim_{c \to 0} \frac{P_{MD}}{c} = 0$.

# Summary

- Developed distributed, non parametric, sequential spectrum sensing algorithms for tackling SNR uncertainty issues.
- Studied the performance (via simulations and asymptotics) of different algorithms at secondary nodes for energy detection and FC MAC.
- Proposed appropriate choice of algorithms for spectrum sensing for the CR system overall.
- Extended the design to handle outliers at the FC receiver.

Thank You!