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PROBABILITY: THEORY AND APPLICATIONS

Workshop on Limit Theorems in Probability

(January 02 - 08, 2013)

DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF SCIENCE

BANGALORE

Workshop on Limit Theorems in Probability

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“Is Countably additivity a must for Limit theorems?”

Rajeeva Karandikar
Chennai Mathematical Institute
Siruseri, Kelambakkam

"Mathematical Population Genetics and Coalescent Processes"

Jason Schweinsberg
University of California,
San Diego, CA

Abstract

In population genetics, we are interested in drawing inferences about the history of a population by examining the DNA in a sample of individuals taken at the present time. Many important problems of this type can be solved by working backwards in time. If we take a sample of size n from a population and follow the ancestral lines backwards in time, then the ancestral lines will coalesce, until eventually the entire sample is traced back to one common ancestor. Under standard assumptions, this process can be modeled by Kingman's coalescent, in which each pair of lineages merges at rate one. However, if some individuals have large numbers of offspring or if natural selection is taking place, then many ancestral lines may merge at approximately the same time, in which case the genealogy of the population is best modeled by a process called a coalescent with multiple mergers. In these lectures, we will begin by discussing the classical theory related to Kingman's coalescent. We will then discuss some models of populations with large family sizes and populations undergoing selection that would lead to genealogies with multiple mergers of ancestral lines. We will discuss what we would expect to see in certain genetic data obtained from a sample of n individuals, such as the number of segregating sites, the allelic partition, and the site frequency spectrum, if the genealogy of the population can be described by Kingman's coalescent. We will also explain how we would expect the data to be affected by multiple mergers of ancestral lines, and we will discuss some of the implications of these results for statistical inference.

“Gibbs Measures and Phase Transitions on Sparse Random Graphs”

Amir Dembo
University of Washington
Seattle, USA

Abstract

Theoretical models of disordered materials are the prototypes of many challenging mathematical problems, with applications ranging from theoretical computer science (random combinatorial problems) to communications (detection, decoding, estimation). This course treats the underlying structure common to many such problems, namely large systems of discrete variables that are strongly interacting according to a mean field model determined by a random sparse graph.

Plan of the course:

1. Introduction: Models on graphs, Phase transitions, Gibbs measures, Mean field equations, Approximation by trees.

2. Potts and independent set models on d-regular graphs: Convergence to the tree measure, Limiting free energy, Applications to Computational Hardness.

3. Factor models on locally tree like graphs: Bethe measures, extremality, belief propagation algorithm and the cavity method. Applications for the Ferromagnetic Ising model.

4. Other related results, open problems and possibly short outline for selected proofs.

References: The short course in workshop is based on following 5 recent ArXiv postings/papers.

1. Factor models on locally tree-like graphs, Amir Dembo, Andrea Montanari, Nike Sun http://arxiv.org/abs/1110.4821 [Lecture 3]

2. The replica symmetric solution for Potts models on d-regular graphs, Amir Dembo, Andrea Montanari, Allan Sly, Nike Sun http://arxiv.org/abs/1207.5500 [Lecture 2]

3. The computational hardness of counting in two-spin models on d-regular graphs, Allan Sly, Nike Sun http://arxiv.org/abs/1203.2602 [Lecture 2]

4. Ferromagnetic Ising Measures on Large Locally Tree-Like Graphs, Anirban Basak, Amir Dembo http://arxiv.org/abs/1205.4749 [Lecture 3]

5. Gibbs Measures and Phase Transitions on Sparse Random Graphs, Amir Dembo, Andrea Montanari http://arxiv.org/abs/0910.5460 [Lecture 1 and likely Lecture 4]

"Concentration inequalities and applications"

Michel Ledoux
Université de Toulouse
Toulouse, France

Abstract

After a brief historical review of limit theorems for Banach space valued random vectors and classical exponential tail inequalities for independent random variables, we present some of the basic ideas and principles of measure concentration. Methods and tools towards concentration inequalities will be emphasized, illustrated with applications to Gaussian samples, combinatorial examples, empirical processes... Some new challenges of measure concentration will be illustrated in the setting of random matrices and their local eigenvalue statistics.

References:

1. S. Boucheron, G. Lugosi, P. Massart. Concentration inequalities: A nonasymptotic theory of independence. Oxford University Press (2013).

2. M. Ledoux. The concentration of measure phenomenon. Mathematical Surveys and Monographs 89, American Mathematical Society (2001).

3. C. McDiarmid. Concentration. In Probabilistic Methods for Algorithmic Discrete Mathematics (ed. M. Habib, C. McDiarmid, J. Ramirez-Alfonsin, and B. Reed), pp. 195–248. Springer, New York (1998).

4. M. Talagrand. Concentration of measure and isoperimetric inequalities in product spaces, Publications I.H.E.S. 81 (1995) 73-205.

"Exchangeability and some applications"

Tim Austin
Courant Institute of Mathematical Sciences
New York, USA

Abstract

Exchangeable sequences of random variables were introduced by De Finetti in the 1930s, and he proved the first theorem describing their structure. In the 1970s and 80s this study was extended to exchangeable models of random partitions (Kingman), two-dimensional arrays (Hoover-Aldous) and later arbitrary higher-dimensional arrays (Kallenberg). Since then exchangeability theory has found several important applications as a means of capturing certain relevant asymptotics of sequence of large finite structures (and some others), including partitions, graphs and metric spaces.

Plan of the course:

Lecture 1: Introduction to exchangeable sequences and arrays and proof of the De Finetti and Aldous-Hoover Theorems.

Lecture 2: Two classical applications of these results:

-- Mass partitions and their role in describing exchangeable random partitions: Kingman's Paintbox Theorem, deduced from De Finetti. Also, introduction to certain

Poisson-Dirichlet processes as a special family of random mass partitions, and the related exchangeable partition process.

-- Random measures on Hilbert space, and their characterization: Dovbysh-Sudakov Theorem, deduced from Aldous-Hoover.

Lecture 3: Introduction to mean-field spin glass models, particularly the Random Energy and Sherrington-Kirkpatrick model. The idea of limit objects as random Hilbert space measures, justified by Dovbysh-Sudakov Theorem. Proof of the asymptotic structure of the Random Energy Model in terms of PD processes, and its relation to the Chinese Restaurant process.

Lecture 4: The role of free energy in spin models. The Aizenman-Sims-Starr scheme giving asymptotic behaviour of the free energy in the Sherrington-Kirkpatrick model in terms of random Hilbert space measures, and the relation to limit objects. Ruelle Probability Cascades as the special class of limit objects that arise from the SK model. Rough description of the Ghirlanda-Guerra identities as a Hilbert-space analog of the Chinese Restaurant process, and (without proof) of the corresponding steps in deducing the structure from this (Panchenko's Theorem).

References:

-- Aldous's St. Flour notes: `Exchangeability and related topics.' In ``Ecole d'´et´e de probabilites de Saint-Flour, XIII—1983, volume 1117 of Lecture Notes in Math., pages 1–198. Springer, Berlin, 1985

-- A survey I wrote a few years ago: http://arxiv.org/abs/0801.1698

A good source for the basic theory of exchangeable random partitions is Bertoin's book ``Random fragmentation and coagulation processes'', Cambridge 2006. Most of that book is about much more sophisticated things that I won't mention.

For spin glasses themselves, the canonical reference is Talagrand's book ``Mean-field models for spin glasses'', Springer, 2010. But actually for the topics I'll discuss, a better place to start might be the Aizenman-Sims-Starr paper: http://arxiv.org/abs/math-ph/0607060

“Tail Inequalities for sums of unbounded random variables”

Ravi Kannan
Microsoft Research Labs, India

Abstract

Martingale and Hoeffding-Chernoff inequalities are widely used. Both (in their common forms) assume absolute bounds on the r.v.'s. The talk is about two tail probability inequalities for sums of (possibly dependent) unbounded random variables in terms of bounds on their moments. We demonstrate the use of the inequalities to prove concentration of several combinatorial quantities like the length of the Travelling Salesman tour. Partial results for matrix-valued random variables are also presented.