Reversible systems, and problems in real and complex analysis having 'hidden dynamics' that is reversible

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Abstract

A reversible map is a special kind of invertible map: it is one that is conjugate to its own inverse, so that the associated dynamical system behaves in the same way when run forwards or backwards in time. We describe this situation by saying that "the past is the future of an alternative present".

Apart from the obvious examples in classical mechanics, one finds that reversible maps arise in many and diverse problems. We shall begin by describing a few examples, where such maps arise in connection with problems of polynomial convexity, complex polynomial approximation, and other areas of functional analysis. These examples motivated our interest in reversibility.

Then we shall formulate the concept of reversibility in its natural abstract context, the theory of groups, and discuss some general features and results.

Finally, we will study reversibility in specific groups of mappings connected to real and complex analysis and geometry, including Möbius groups, groups of formal power series, groups of biholomorphic maps, homeomorphism groups and diffeomorphism groups.

There are many open questions about reversibility, and about the related subjects of involutions and products of involutions. We will describe some of these.