ON AN IRREDUCIBILITY PROPERTY OF IRREDUCIBLE CHARACTERS OF SIMPLE LIE ALGEBRAS

The goal of these lectures is to establish an irreducibility property for the characters of finite dimensional, irreducible representations of simple Lie algebras (or simple algebraic groups) over the complex numbers, i.e., that the characters of irreducible representations are irreducible after dividing out by (generalized) Weyl denominator type factors.

One consequence is that it establishes an unique factorization of tensor products of irreducible representations of simple Lie algebras, similar to the unique factorization of an integer into primes.

For $GL(r)$ the irreducibility result can be stated as follows: let $\lambda = (a_1 \geq a_2 \geq \cdots a_{r-1} \geq 0)$ be a r-tuple of relatively prime non-negative integers. Then the polynomial function

$$\det(x_i^{da_j})/\det(x_i^{d(r-j)})$$

is irreducible over the complex numbers for any natural number d.

In my first lecture, I will outline the motivations behind the above result and consider the problem of recovering a representation knowing that some tensor, symmetric or exterior power of the original represetations coincide. For example, it is well known that the character determines the representation of a finite group or a continuous finite dimensional representation of a compact group. For semisimple representations of a general group, by an algebraic trick this can be reduced to that of compact groups. Similar algebraic methods allow us to handle the above problem.

In the subsequent lectures, I will talk about the irreducibility property of characters.