E0 219 Linear Algebra and Applications / August-December 2016

(ME, MSc. Ph. D. Programmes)

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Lectures : Monday and Wednesday ; 11:00–12:30 **Venue:** CSA, Lecture Hall (Room No. 117)

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Midterms : 1-st Midterm : Saturday, September 17, 2016; 15:00 – 17:00 **2-nd Midterm :** Saturday, October 22, 2016; 15:00 – 17:00

Final Examination : December ??, 2016, 09:00 – -12:00

Evaluation Weightage : Assignments : 20% **Midterms (Two) :** 30% **Final Examination :** 50%

Range of Marks for Grades (Total 100 Marks)						
	Grade S	**Grade A**	**Grade B**	**Grade C**	**Grade D**	**Grade F**
Marks-Range	> 90	76 — 90	61 — 75	46 — 60	35 — 45	< 35

	Grade A$^+$	**Grade A**	**Grade B$^+$**	**Grade B**	**Grade C**	**Grade D**	**Grade F**
Marks-Range	> 90	81 — 90	71 — 80	61 — 70	51 — 60	40 — 50	< 40

8. Quotient spaces

Submit a solution of the $*$-Exercise ONLY. **Due Date :** Monday, 03-10-2016 (Before the Class)

In the following Exercises K denote a field and V denote a K-vector space.

8.1 Let $n \in \mathbb{N}$. A subspace U of the vector space V has the codimension n if and only if there exist n linearly independent forms $f_1, \ldots, f_n$ on V with $U = \bigcap_{i=1}^n \operatorname{Ker} f_i$.

8.2 Let $U_1, \ldots, U_n$ be finite-codimensional subspaces of V and $U := \bigcap_{i=1}^n U_i$ be their intersection. Further, let $U_i' := \bigcap_{j \neq i} U_j$, $i = 1, \ldots, n$.

(a) Show that U is finite-codimensional with $\operatorname{Codim}_K(U, V) \leq \sum_{i=1}^n \operatorname{Codim}_K(U_i, V)$.

(b) The following statements are equivalent :

(i) The inequality in part (a) is equality.

(ii) The canonical homomorphism $V/U \to \bigoplus_{i=1}^n V/U_i$ is an isomorphism.

(iii) $U_i + U_i' = V$ for $i = 1, \ldots, n$.

(iv) $U_1' + \cdots + U_n' = V$.

(v) The sum U_i°, $i = 1, \ldots, n$, of subspaces U_i°, $i = 1, \ldots, n$, in V^* is direct.

*__8.3__ Let $f : V \to W$ be a K-linear map of finite dimensional K-vector spaces. Then show that :

$$\operatorname{Dim}_K \operatorname{Ker} f - \operatorname{Dim}_K \operatorname{Coker} f = \operatorname{Dim}_K V - \operatorname{Dim}_K W .$$

(**Remark :** See also Supplement S8.2 (b) and Supplement 8.4 (b).)

8.4 Let U, W be subspaces of V with $U \subseteq W$. If W' is a complement of W in V, then $(U + W')/U$ is a complement of W/U in V/U which is isomorphic to W'.

8.5 Let f be a K-linear operator on a K-vector space V. Then show that the following statements are equivalent :

(i) f induces an automorphism of $\operatorname{Im} f$.

(ii) f induces an automorphism of $V / \operatorname{Ker} f$.

(iii) $V = \operatorname{Im} f \oplus \operatorname{Ker} f$.

(iv) $\operatorname{Ker} f$ has a f-invariant complement W such that the restriction $f | W$ is an automorphism of W. (For the finite dimensional case, see Supplement S6.20 — The K-subspace in (iv) is necessarily $\operatorname{Im} f$.)