MA 313 Algebraic Number Theory / January-April 2016

(Int PhD. and Ph. D. Programmes)

Tel : +91-(0)80-2293 3212/(CSA 2239)   **E-mails :** patil@math.iisc.ernet.in / dppatil@csa.iisc.ernet.in

Lectures : Monday and Wednesday ; 15:30–17:00   **Venue:** MA LH-1 / LH-3 **(if LH-1 is not free)**

Midterms : Thursday, Feb 18, 2016, 10:00–11:30   **Seminars :** Fri April 15, Sat April 16, 2016, 15:00–17:00

Final Examination : Saturday, April 23, 2016, 14:00–17:00

Evaluation Weightage : Seminar : 20%   **Midterms :** 30%   **Final Examination :** 50%

Range of Marks for Grades (Total 100 Marks)						
	Grade S	**Grade A**	**Grade B**	**Grade C**	**Grade D**	**Grade F**
Marks-Range	> 90	76–90	61–75	46–60	35–45	< 35

MID TERM

Monday, February 18, 2016   **10:00 to 11:30**   **Maximum Points :** 30 Points

MT.1 Let A be a commutative ring.

(a) Let $f : V \to V$ be an endomorphism of a noetherian A-module V. Show that if f is surjective, then f is bijective. [5 points]

(b) Suppose that A is a noetherian ring and $\mathfrak{a} \subseteq A$ is a non-zero ideal A. Show that the rings A and $A/\mathfrak{a}$ are not isomorphic. [5 points]

MT.2 Let $A \subseteq B$ be an extension of rings.

(a) (L e m m a o f G a u s s) If $f, g \in B[X]$ are monic polynomials with $fg \in A[X]$, then show that coefficients of f and g are integral over A. [3 points]
(**Hint :** let C be a ring extension of B such that both f and g spilits into linear factors in $C[X]$. Then the zeros of f and g are integral over A(why?).)

(b) Suppose that A and B are integral domains with quotient fields K and L respectively. If A is integrally closed and if $x \in B$ is integral over A. Show that the minimal polynomial $\mu_{x,K}$ has coefficients in A and $\mu(x) = 0$ is the minimal integral equation of x over A. Further, show that $A[x]$ is a free A-module of rank $\deg \mu_{x,K} = [K(x) : K]$. (**Hint :** Use part (a).) [7 points]

MT.3 Let $\mathbb{A}$ be the ring of algebraic integers in $\mathbb{C}$.

(a) Show that $\mathbb{A}$ is not noetherian and there are no prime elements in $\mathbb{A}$. [4 points]

(b) Let $\mathbb{P}$ denote the set of prime numbers. Show that for every maximal ideal $\mathfrak{m} \in \operatorname{Spm} \mathbb{A}$, there is a unique prime number $p \in \mathbb{P}$ with $\mathfrak{m} \cap \mathbb{Z} = \mathbb{Z}p$. Further, show that the map $\varphi : \operatorname{Spm} \mathbb{A} \to \mathbb{P}$, $\mathfrak{m} \mapsto p$, is surjective. Is the map φ is injective? [6 points]

G o o d   L u c k