A_AG - 05 Algebra, Arithmetic and Geometry – With a View Toward Applications / 2005

Lectures : Tuesday/Thursday 18:15–19:15 ; LH-1, Department of Mathematics

Supplementary Lectures : Friday 18:15–19:15 ; LH-1, Department of Mathematics

Topics (January 2006 — June 2006)

◇ Algebra --- Rings, Modules and Algebras

- **Rings :**
  - Polynomial rings, Zeros of polynomials, Resultants and Discriminants, Gröbner basis, Hilbert basis theorem. Euclidean rings, Principal ideal domains and Factorial rings. Factorisation in rings.
  - Elementary symmetric functions and Fundamental theorem on symmetric functions. Proof of Fundamental theorem of Algebra.
  - Prime ideals and Maximal ideals — Chinese remainder theorem.
  - Noetherian rings and Modules, Graded rings and modules, Formal power series rings.
  - Local rings, Nakayamma-lemma. Localisation, Primary decomposition, Integral extensions.
- **Modules :**
  - Module Homomorphisms — Basic theorems, Exact sequences.
  - Projective and Injective modules.
  - Hom and Duality. Tensor Products. Multilinear algebra.
  - Modules over PID. Jordan Cannonical form.
- **Algebras :**
  - Finite algebras over a field — Finite dimensional division algebras.
  - Finite type algebras over a field — Noether's normalisation lemma.

◇ Arithmetic --- Algebraic extensions, Galois theory and Algebraic number theory

- **Galois Theory :**
  - Finite and Algebraic extensions. Algebraic closure, Algebraically closed fields.
  - Splitting fields, Normal extensions. Separable and inseparable extensions. Primitive elements. Finite fields.
  - Galois Extensions. The fundamental theorem of Galois theory, Galois groups of polynomials. Solvable and radical extensions. Insolvability of the Quintic, Computation of Galois groups over the field of rational numbers.
  - Norm and Trace. Roots of unity, Cyclotomic extensions. Cyclic and Abelian extensions.
  - Normal basis theorem.
  - Infinite Galois extensions.
- **Algebraic Number Theory :**
  - Integral extensions — Integral closure. Algebraic Number Fields and Algebraic integers.
  - Resultants and Discriminants. Integral Bases.
  - Discrete valuation rings and Dedekind domains.
  - Lattices — Minkowski theory. The class number, Finiteness of class number, Dirichlet's unit theorem,
  - Differents and Discriminants.

◇ Geometry --- Basic Algebraic Geometry

- Affine varieties, Algebra-Geometry Dictionary : Various forms of Hilbert's Nullstellensatz, Ideal-Variety correspondence, Irreducible varieties and Prime ideals, Decomposition of a variety into irreducibles.
- The prime spectrum of a ring — Affine schemes.
- Projective algebraic geometry — Projective varieties, Bezout's theorem, Elliptic curves.
- The Dimesion of a variety — The Hilbert function and the Dimension, Elementary properties of Dimension, Dimension and algebraic independence, Dimension and non-singularity.

◇ In Supplementary lectures on Fridays ---

- **Groups :** Groups actions — Sylow theorems. Permutations groups, Alternating groups, Special linear groups. Solvable and Nilpotent groups.
- **Linear Algebra** (over commutative rings) **:** Theory of Determinants — Basic theorems on determinants, Affine and Projective Geometry.

Topics for Seminars by Participants (January 2006 — June 2006)

- Sylow theorems.
- Structure of the unit group $(\mathbb{Z}_m)^\times$ of $\mathbb{Z}_m$.
- Quadratic reciprocity.
- Free groups and the theorem of Nielsen and Schreier.
- Divisible abelian groups.
- Fermat's two square theorem.
- Lagrange's four square theorem.
- Matrix rings.
- Semi-simple rings and modules.
- Pell's Equation and Continued fractions.

Texts/References

• Algebra :

[1] **Artin, M.,** *Algebra*, Prentice-Hall, 1994.

[2] **Herstein, I.N.,** *Topics in Algebra*, Wiley Eastern, 1987.

[3] **Jacobson, N.,** *Basic Algebra*, Vols. I & II, Hindustan Pub. Co., 1984.

[4] **Lang, S.,** *Algebra*, Third edition, Addison-Wesley, 1993.

[5] **Hungerford, T. W.,** *Algebra*, Graduate Texts in Mathematics 73, Springer-Verlag, 1974.

[6] **van der Waerden, B. L.,** *Algebra* I, II, Heidelberger Taschenbücher 12 und 23, Springer-Verlag, Berlin-Heidelberg-New York, 1971.

• Commutative Algebra :

[7] **Atiyah, M. F.** and **Macdonald, I. G.,** *Introduction to Commutative Algebra*, Addison-Wesley, 1969.

[8] **Eisenbud, D.,** *Commutative Algebra with a View Toward Algebraic Geometry*, Springer-Verlag, 1995.

[9] **Serre, J.-P.,** *Local Algebra* (Translated from French), Springer Monographs in Mathematics, Springer-Verlag, 2000.

[10] **Zariski, O.** and **Samuel, P.,** *Commutative Algebra*, Vols. I & II, Van Nostrand, 1958 and 1960.

• Algebraic Number Theory :

[11] **Borevich, Z. I. and Shafarevich, I. R.,** *Number Theory*, Academic Press Inc. New York, 1966.

[12] **Hasse, H.,** *Vorlesungen über Zahlentheorie*, Springer-verlag, Berlin, 1964.

[13] **Ireland, K. and Rosen, M.,** *A Classical Introduction to Modern Number Theory*, Graduate Studies in Mathematics, Volume 84, Springer-Verlag, 1990.

[14] **Lorenzini, D.,** *An Invitation to Alrithmetic Geometry*, Graduate Studies in Mathematics, Volume 9, American Mathematical Society, 1996.

[15] **Neukirsch, J.,** *Algebraic Number Theory*, Grundleheren der mathematischen Wissenschaften, Volume 322, Springer-Verlag, 1991.

• Algebraic Geometry :

[17] **Abhyankar, S. S.,** *Algebraic Geometry for Scientists and Engineers*, American Mathematical Society, 1990.

[18] **Cox, D., Little, J. and O'Shea, D.,** *Ideals, Varieties and Algorithms*, Undergraduate Texts in Mathematics, Springer-Verlag, 1996.

[19] **Cox, D., Little, J. and O'Shea, D.,** *Using Algebraic Geometry*, Graduate Texts in Mathematics, Volume 185, Springer-Verlag, 1998.

[20] **Fulton, W.,** *Algebraic Curves*, Benjamin, 1969.

[21] **Hartshorne, R.,** *Algebraic Geometry*, Graduate Texts in Mathematics 52, Springer-Verlag, 1977.

[22] **Patil, D. P. and Storch, U.,** *Introduction to Algebraic Geometry and Commutative Algebra*, *Anamaya Publishers*, ISBN 81-88342-61-0 (co-publishing and worldwide distribution by M/s Anshan Ltd., U.K.).

[23] **Shafarevich, I. R.,** *Basic Algebraic Geometry*, Springer-Verlag, 1974.