# Basic Algebra
# D. P. Patil

I will be offering a three month course entitled '' **Basic Algebra**'' from May to July 2003. This course is intended for all course and research students who use algebraic techniques in their work. It will ONLY presuppose a basic knowledge of sets and groups. Depending on the interests of the participants, special topics other than those mentioned below could also be covered. This course is meant to prepare students for the study of *Commutative Algebra and Algebraic Geometry*.

- **1. Rings and Modules**
    - Rings, Subrings, Prime rings, Integral domains and fields.
    - Modules, Submodules, Ideals, Algebras.
    - Prime and maximal ideals, Nil and Jacobson radicals.
    - Linear Equations, Linear dependence, Free modules, Rank of free modules.
    - Homomorphisms of rings, Homomorphisms of modules. Exact Sequences.
    - Hom and Tensor products. Duality, Projective, injective and flat modules.
    - Rings and Modules with chain conditions, Modules of finite lengths.
    - Multilinear, alternating maps. Determinants and Volume. Norms in Algebras.

- **2. Factorisation in rings**
    - Rings and modules of fractions.
    - Polynomial rings, Zeros of polynomials, Resultants and Discriminants, Finite algebras over a field.
    - Algebraic and integral elements, Algebraic closures, Integral closures.
    - Euclidean rings, Principal ideal domains and Factorial rings. Prime factorisation in Polynomial rings.
    - Modules over PIDs.
    - Graded rings and modules.
    - Formal power series rings.

- **3. Linear Operators**
    - Characteristic and Minimal polynomials, Decomposition theorems, Triangulable, Diagonalizable, Semi-simple and seperable operators, Jordon cannocial form.
    - Bilinear and sesquilinear forms, Symmetric and complex-hermitian forms. Type of hermitain forms.
    - Inner product spaces, Linear and affine isometries. Self-adjoint and Normal operators, Principal axix theorem.
    - Classical groups.

- **4. Galois Theory**
    - Spliting fields, Seperable polynomials.
    - Seperable algebras over a field. Trace form.
    - Galois Extensions. The fundamental theorem of Galois theory.
    - Galois groups of polynomials. Solvable and radical extensions. Insolvability of the Quintic.
    - Computation of Galois groups over $\mathbb{Q}$ .

- **5. Commutative algebra and Algebraic geometry** —- **Preliminaries**
    - Noetherian rings and modules, Hilbert-basis theorem, Affine algebraic sets.
    - Affine varieties, The prime spectrum of a ring.
    - Various forms of Hilbert's Nullstellensatz.

● ● **Prerequisites :** Elementary abstract algebra and Linear algebra.

## Time-table

| Day | Time | Venue |
|---|---|---|
| **Monday** | **11:30 AM to 1:00 PM** | Lecture Hall-I, Department of Mathematics |
| **Wednesday** | **11:30 AM to 1:00 PM** | Lecture Hall-I, Department of Mathematics |
| **Friday** | **11:30 AM to 1:00 PM** | Lecture Hall-I, Department of Mathematics |

● ● The first lecture will be on **Friday, May 2, 2003**.

**D. P. Patil**

**Basic Algebra** —— **(May-July 2003)**
**Lectures :** Monday/Wednesday/Friday 11:30 AM–1:00 PM

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# Texts/References

(* references will be used more often.)

● **Algebra :**

[1]  **Artin, M.,** *Algebra*, Prentice-Hall, 1994.

[2]  **Herstein, I.N.,** *Topics in Algebra*, Wiley Eastern, 1987.

*[3]  **Jacobson, N.,** *Basic Algebra*, Vols. I & II, Hindustan Pub. Co., 1984.

[4]  **Lang, S.,** *Algebra*, Third edition, Addison-Wesley, 1993.

*[5]  **Hungerford, T. W.,** *Algebra*, Graduate Texts in Mathematics 73, Springer-Verlag, 1974.


● **Commutative Algebra :**

*[6]  **Atiyah, M. F.** and **Macdonald, I. G.,** *Introduction to Commutative Algebra*, Addison-Wesley, 1969.

[7]  **Eisenbud, D.,** *Commutative Algebra with a View Toward Algebraic Geometry*, Springer-Verlag, 1995.

[8]  **Matsumura, H.,** *Commutative Algebra*, W. A. Benjamin Co., New York, 1970.

[9]  **Raghavan, S., B. Singh** and **Sridharan, R.,** *Homological Methods in Commutative Algebra*, TIFR Mathematical Pamphlet Number 5, Oxford University Press, 1977.

*[10]  **Serre, J.-P.,** *Local Algebra* (Translated from French), Springer Monographs in Mathematics, Springer-Verlag, 2000.

*[11]  **Zariski, O.** and **Samuel, P.,** *Commutative Algebra*, Vols. I & II, Van Nostrand, 1958 and 1960.


● **Algebraic Geometry :**

[12]  **Abhyankar, S. S.,** *Algebraic Geometry for Scientists and Engineers*, American Mathematical Society, 1990.

*[13]  **Fulton, W.,** *Algebraic Curves*, Benjamin, 1969.

[14]  **Harris, J.,** *Algebraic Geometry: A First Course*, Springer-Verlag, 1992.

*[15]  **Hartshorne, R.,** *Algebraic Geometry*, Graduate Texts in Mathematics 52, Springer-Verlag, 1977.

*[16]  **Shafarevich, I. R.,** *Basic Algebraic Geometry*, Springer-Verlag, 1974.

[17]  **Walker, R. J.,** *Algebraic Curves*, Springer-Verlag, 1950.