E0 219 Linear Algebra and Applications / August-December 2011

(ME, MSc. Ph. D. Programmes)

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Tel : +91-(0)80-2293 2239/(Maths Dept. 3212) **E-mails :** dppatil@csa.iisc.ernet.in / patil@math.iisc.ernet.in

Lectures : Monday and Wednesday ; 11:30–13:00 **Venue:** CSA, Lecture Hall (Room No. 117)

Corrections by : **Jasine Babu** (jasinekb@gmail.com) / **Nitin Singh** (nitin@math.iisc.ernet.in) / **Amulya Ratna Swain** (amulya@csa.iisc.ernet.in) / **Achintya Kundu** (achintya.ece@gmail.com)

1-st Midterm : Saturday, September 17, 2011; 15:00 -17:00 **2-nd Midterm :** Saturday, October 22, 2011; 10:30 -12:30

Evaluation Weightage : Assignments : 20% **Midterms (Two) :** 30% **Final Examination :** 50%

TEST 1

Saturday, September 17, 2011 **15:00 to 17:00** **Maximum Points :** 20 Points

- **Attempt any FOUR Questions.**

T1.1 (a). For two elements $a,b \in K$ in a field K, determine the conditions on a,b so that $(a,b),(b,a) \in K^2$ form a basis of the K-vector space K^2. [2 points]

(b). Let p_1,p_2,p_3 be three distinct prime numbers. Show that the square-roots $\sqrt{p_1},\sqrt{p_2},\sqrt{p_3}$ are linearly independent over $\mathbb{Q}$. [3 points]

T1.2 Let U be the $\mathbb{R}$-subspace (of the $\mathbb{R}$-vector space $\mathbb{R}^{\mathbb{R}}$ of all real-valued functions on $\mathbb{R}$) generated by the functions $\cos^n t\,, n \in \mathbb{N}$, i. e. $U = \sum_{n\in\mathbb{N}} \mathbb{R}\cos^n t$. Further, let W be the $\mathbb{R}$-subspace (of the $\mathbb{R}$-vector space $\mathbb{R}^{\mathbb{R}}$ of all real-valued functions on $\mathbb{R}$) generated by the functions $\cos nt\,, n\in\mathbb{N}$, i. e. $W = \sum_{n\in\mathbb{N}} \mathbb{R}\cos nt$. Show that $U = W$. [5 points]

(**Hint :** Use the addition formula $\cos(x+y) = \cos x\cdot\cos y - \sin x\cdot\sin y$ for all $x,y\in\mathbb{R}$.)

T1.3 Let $n\in\mathbb{N}^*$ and let $\mathbb{K}[t]_{2n}$ (respectively, $\mathbb{K}[t]_n$) denote the $\mathbb{K}$-vector space of all polynomials (over $\mathbb{K}$) of degree $< 2n$ (respectively, $< n$). Further, let

$$T:\mathbb{K}[t]_{2n}\to\mathbb{K}[t]_n\ ,\ f\mapsto T(f) := \sum_{k=0}^{n-1}\frac{f^{(k)}(1)}{k!}(t-1)^k\,,$$ where $f^{(k)}$ denote the k-th derivative of f,

be the map which maps every polynomial $f\in K[t]_{2n}$ to its *Taylor-polynomial* of degree $< n$ of f at 1. Show that T is $\mathbb{K}$-linear. Determine $\operatorname{Ker} T$ and $\operatorname{Im} T$. What are $\operatorname{Dim}_{\mathbb{K}}\operatorname{Ker} T$ and $\operatorname{Dim}_{\mathbb{K}}\operatorname{Im} T$? Justify your answers. [5 points]

T1.4 Let V be a K-vector space of dimension $n\in\mathbb{N}$ and let $U\subseteq V$ be a subspace of codimension $r(=\operatorname{Dim}_K V - \operatorname{Dim}_K U)$ in V. Show that there exist r hyper-planes (subspaces of codimension 1) $H_1,\ldots,H_r$ in V such that $U = H_1\cap\cdots\cap H_r$. [5 points]

T1.5 Let $f:V\to V$ be an operator on the finite dimensional K-vector space V. Show that the following statements are equivalent :

(i) $\operatorname{Ker} f = \operatorname{Im} f$. (ii) $f^2 = 0$ and $\operatorname{Dim}_K V = 2\cdot\operatorname{Rank} f$. [5 points]

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