MA-219 Linear Algebra

Test 1

Saturday, September 06, 2003 ; 10:00 AM - 11:30 AM

Let K denote a field.

T1.1. Show that

a). the family $\{f_a \mid a \in \mathbb{R}\} \subseteq \mathbb{R}^{\mathbb{R}}$ is linearly independent over $\mathbb{R}$, where $f_a : \mathbb{R} \to \mathbb{R}$ is the function defined by $t \mapsto |t-a|$; [2 Points]

b). the family $\{t^n e^{at} \mid n \in \mathbb{N}\,,\ a \in \mathbb{C}\}$ is linearly independent over $\mathbb{C}$. [3 Points]

T1.2. Let V be a vector space over K.

a). If V has a countable infinite basis, then show that every K-subspace U of V has a countable basis. [2 Points]

b). Let U and W be two subspaces of V with bases $x_i\,,\ i \in I$ and $y_j\,,\ j \in J$ respectively. Show that $x_i\,,\ y_j\,,\ (i,j) \in I \times J$ is a basis of $U + W$ if and only if $U \cap W = 0$. [3 Points]

T1.3. Suppose that K has at least n elements and $d := \mathrm{Dim}_K V \in \mathbb{N}^+$. Let $U_1, \ldots, U_n$ be subspaces of V of equal dimension r and let $u_{1i}, \ldots, u_{ir}$ be a basis of U_i for $i = 1, \ldots, r$. Show that there exist $d-r$ vectors $x_1, \ldots, x_{d-r} \in V$ such that $u_{1i}, \ldots, u_{ir}\,,\ x_1, \ldots, x_{d-r}$ is a basis of V for every $i = 1, \ldots, n$. [5 Points]

T1.4. Let E be an affine space over $\mathbb{R}$ and assume that $\mathrm{Dim}\, E \geq n \in \mathbb{N}^+$. Let $(P_0, \ldots, P_n)$ be an n-simplex in E and let S be the center of mass of the points $P_0, \ldots, P_n$ with equal weights 1. Show that

a). $\overrightarrow{P_0S} = \dfrac{1}{n+1} \displaystyle\sum_{i=0}^{n} \overrightarrow{P_0P_i}$. [2 Points]

b). Assume that $n \geq 2$. For $i = 0, \ldots, n$, let S_i be the center of mass of the $(n-1)$- simplex obtained from the n-simplex $(P_0, \ldots, P_n)$ by removing the point P_i. Show that the affine lines $P_iS_i\,,\ i = 0, \ldots, n$ intersect at the point S and it divide each affine line P_iS_i in the equal ratio $n : (n+1)$ i.e., $\overrightarrow{P_iS} = \dfrac{n}{(n+1)} \overrightarrow{P_iS_i}$ for all $i = 0, \ldots, n$. [3 Points]