

UM 101 HOMEWORK ASSIGNMENT 11

Posted on January 12, 2023
(NOT FOR SUBMISSION)

- These problems are for self-study.
- Some of these problems will be discussed at the next tutorial.
- A 15-minute quiz based on the topics of this assignment will be conducted at the end of the tutorial section.

Problem 1. Problems 1-20 from Section 5.8 in Apostol.

Problem 2. Recall the definition of the function $\ln : \mathbb{R} \rightarrow \mathbb{R}$ from Lecture 33. Prove that

- (a) $\ln(xy) = \ln(x) + \ln(y)$ for all $x, y > 0$,
- (b) $\int \frac{dt}{t} = \ln|t| + C$ for all $t \neq 0$,
- (c) \ln is one-to-one and onto \mathbb{R} .

Problem 3. Recall the definition of $\exp : \mathbb{R} \rightarrow (0, \infty)$. Show that

$$\exp(r) = e^r$$

for every rational number r . Recall that if $r = m/n$, where $m \in \mathbb{Z}$ and $n \in \mathbb{P}$, then e^r is the n^{th} -root of e^m .

Problem 4. Now that we have a definition for $f(x) = x^r$ for any real $x > 0$ and $r \in \mathbb{R}$, prove that f is differentiable and

$$f'(x) = rx^{r-1}, x > 0,$$

for an real number r .

Problem 5. Problem 40 from Section 6.17 in Apostol.