UM 101 HOMEWORK ASSIGNMENT 12

Posted on January 20, 2023
(NOT FOR SUBMISSION)

- These problems are for self-study.
- Some of these problems will be discussed at the next tutorial.

Problem 1. Problem 29 in Section 5.5 of Apostol.

Problem 2. Let $\mathscr{S}$ denote the vector space of all real sequences as defined in class. Determine which of the following subsets of $\mathscr{S}$ are subspaces.

(a) $S =$ the set of all convergent real sequences.
(b) $S =$ the set of all divergent real sequences.
(c) $S = \big\{ \{a_j\}_{j\in\mathbb{N}} : \text{infinitely many } a_j's \text{ are } 0 \big\}$.

Problem 3. Recall the $P_{\leq d}$ is the space of real polynomials of degree at most d. Determine whether or not

$$P_{\leq 2} = \operatorname{span}(1 + x, x + x^2, 1 + x^2).$$

Problem 4. Let V be a vector space over a field F. Let $S \subseteq V$. Show that span(S) is the intersection of all the subspaces of V containing S, i.e.,

$$\operatorname{span}(S) = \bigcap_{\substack{W \text{ is a subspace of } V \\ S\subseteq W}} W.$$

Problem 5. Let V be a vector space over a field F. Determine whether the following statements are true (provide a proof) or false (provide a counterexample).

(a) $\operatorname{span}(S \cup T) = \operatorname{span}(S) \cup \operatorname{span}(T)$.
(b) $\operatorname{span}(S \cap T) = \operatorname{span}(S) \cap \operatorname{span}(T)$.

Problem 6 (extra food for thought). Let $F = \mathbb{C}$, endowed with complex addition and complex multiplication (as defined in Lecture 33). Let $V = \mathbb{R}$ and $+$ be the usual addition on real numbers. Show that there is no scalar product '$\cdot$' such that

$$(r + i0) \cdot v = rv, \qquad \forall r, v \in \mathbb{R},$$

and $(\mathbb{R}, +, \cdot)$ is a field over F.