UM 101: QUIZ 2

Nov. 03, 2022

Duration. 10 minutes

Maximum score. 10 points

Name.

Tutorial section.

Problem. Let $(F, +, \cdot)$ be a field. Suppose there exists a subset $P \subseteq F$ which satisfies the following three properties:

(1) If $x, y \in P$, then $x + y \in P$ and $x \cdot y \in P$.

(2) For every $x \in F$ such that $x \neq 0$, either $x \in P$ or $-x \in P$, but not both.

(3) $0 \notin P$.

Show that there is a relation $<$ on F such that $(F, +, \cdot, <)$ is an ordered field.

You are free to use (without remark) any properties of a field that were either stated in class or are covered in Theorems 1.1-1.15 of Apostol.