

UM 101: QUIZ 8

Jan. 05, 2023

Duration. 15 minutes

Name.

Maximum score. 10 points

Tutorial section.

Problem. Let $a < b$ be real numbers. Let $f : [a, b] \rightarrow \mathbb{R}$ be a bounded function. Suppose, for every $\varepsilon > 0$, there exist step functions $s_\varepsilon, t_\varepsilon : [a, b] \rightarrow \mathbb{R}$ such that

(i) $s_\varepsilon(x) \leq f(x)$ for all $x \in [a, b]$,

(ii) $t_\varepsilon(x) \geq f(x)$ for all $x \in [a, b]$, and

(iii) $\int_a^b t_\varepsilon(x) dx - \int_a^b s_\varepsilon(x) dx < \varepsilon$.

Show that f is Riemann integrable on $[a, b]$. You may use any theorems stated in class, as long as you cite them in your write-up.