

UM 101: QUIZ 9

Jan. 12, 2023

**Duration.** 15 minutes

**Name.**

**Maximum score.** 10 points

**Tutorial section.**

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**Problem.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be continuous functions such that

$$\int_0^x f(t)dt = x + \int_0^x g(t)dt \quad \forall x \in \mathbb{R}.$$

Prove that if  $f$  is differentiable on  $\mathbb{R}$ , then so is  $g$ , and  $f'(x) = g'(x)$  for all  $x \in \mathbb{R}$ . Please provide a **rigorous** argument. If you use any theorems stated in class, you **must justify** why they apply to your situation.