GEOPHYSICAL RESEARCH LETTERS, VOL. 24, NO.10, PAGES 1239-1242, MAY 15, 1997

A climate predictability index and its applications

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Abstract. In this letter, a climate predictability index is proposed. It quantifies the predictability of three major components comprising the climate — temperature, pressure and precipitation. The quantification is done using a fractal dimensional analysis of the corresponding time series. The climate predictability index is calculated for 25 stations spread throughout India using the Global Historical Climatology Network dataset. Change in the index with the seasons suggests a strong influence of more than one climatic dynamics. In such cases, calculations done using mean yearly data are suspect. The index is shown to be useful in studying the interplay between various climatic components (viz. temperature, pressure and precipitation). Changes in predictability indices for temperature and pressure are seen to affect the index for precipitation. The index can be used as a discriminant for determining which stations are selected for use in developing regional climatic models.

1. Introduction

Fractal dimensional analysis of geophysical time series has a long history [*Hurst et. al.*, 1965; *Mandelbrot and Wallis*, 1969; *Fluegeman and Snow*, 1989; *Hsui et. al.*, 1993; *Turcotte*, 1992]. However, these analyses have concentrated on obtaining the fractal dimensions for individual time series. In this letter, we attempt to go one step further and link these dimensions to the dynamics of the climate. In particular, we want to study how these fractal dimensions are linked together (since the underlying processes are dynamically interlinked).

We start by defining a climate predictability index. This index comprises predictability indices for three major components of climate — temperature, pressure and precipitation. These indices are obtained using a fractal dimensional analysis of the times series for these three processes as explained in the next section. Next, the analysis is applied to climatic data from 25 stations spread throughout India. Use of the predictability index in analysing the above data is demonstrated. We conclude with a discussion of our results.

Paper number 97GL01058.
0094-8534/97/97GL-01058$05.00

2. A Climate Predictability Index

Climate over any continent largely comprises of four major components — geographic parameters, temperature, pressure and precipitation. Geographic parameters (latitude, longitude, distance from the sea and elevation) are however constant for a given location. We therefore concentrate on the other three dynamic components. Time series for the three variables (temperature, pressure and precipitation) have already been shown [*Hurst et. al.*, 1965; *Mandelbrot and Wallis*, 1969; *Fluegeman and Snow*, 1989; *Hsui et. al.*, 1993; *Turcotte*, 1992] to correspond to a fractional Brownian motion [*Mandelbrot*, 1983].

One can understand fractional Brownian motion as follows. Consider a discrete time series specified by $(t_i, x(t_i))$ $(i = 1, 2, \ldots, N)$. Here t denotes the time and x the amplitude of the variable under consideration (either temperature, pressure or precipitation in our case). For a fractional Brownian motion, the amplitude increments $x(t_j) - x(t_i)$ have a Gaussian distribution with variance [*Voss*, 1985]:

$$< [x(t_j) - x(t_i)]^2 > \sim (t_j - t_i)^{2H}, \tag{1}$$

where the brackets $<>$ denote the average over many samples of $x(t)$. The parameter H is called the Hurst exponent and takes values between 0 and 1. If $H = 0.5$, we obtain the usual Brownian motion. The Hurst exponent is related to the fractal dimension D of the time series curve by the formula [*Voss*, 1985]

$$D = 2 - H. \tag{2}$$

If the fractal dimension D for the time series is 1.5, we again get the usual Brownian motion. In this case, there is no correlation between amplitude changes corresponding to two successive time intervals. Therefore, no trend in amplitude can be discerned from the time series and hence the process is unpredictable. However, as the fractal dimension decreases to 1, the process becomes more and more predictable as it exhibits "persistence". That is, the future trend is more and more likely to follow an established trend [*Hsui et. al.*, 1993].

As the fractal dimension increases from 1.5 to 2, the process exhibits "anti-persistence". That is, a decrease in the amplitude of the process is more likely to lead to an increase in the future. Hence, the predictability again increases. However, we will be concerned only with persistence behaviour since all geophysical time records analysed till date [*Hurst et. al.*, 1965; *Mandelbrot and Wallis*, 1969; *Fluegeman and Snow*, 1989; *Hsui et. al.*, 1993; *Turcotte*, 1992] exhibit this behaviour.

We obtain the fractal dimensions of the time series corresponding to temperature, pressure and precipitation for a given location. The fractal dimensions are denoted by D_T, D_P and D_R respectively. These are obtained using the rescaled range (R/S) analysis [*Mandelbrot and Wallis*, 1969]. In R/S analysis, one first calculates the ratio R/S where R is the cumulated range of the process over different time intervals with the sample average removed and S is the square root of the sample variance. The R/S value is averaged over all time intervals with the same period. The slope of the log-log plot of the average R/S values versus time intervals gives the Hurst exponent H which in turn is related to the fractal dimension D through the formula given earlier.

The R/S analysis is used merely because it has been the conventional technique used for geophysical time records [*Hurst et. al.*, 1965; *Mandelbrot and Wallis*, 1969; *Fluegeman and Snow*, 1989; *Hsui et. al.*, 1993; *Turcotte*, 1992]. Any other method would be equally adequate. Predictability indices (denoted by PI_T, PI_P and PI_R respectively) for temperature, pressure and precipitation are defined as follows:

$$\begin{aligned} \mathrm{PI}_T &= 2|D_T - 1.5|; \quad \mathrm{PI}_P = 2|D_P - 1.5|; \\ \mathrm{PI}_R &= 2|D_R - 1.5|. \end{aligned} \tag{3}$$

Here $|D|$ denotes the absolute value of the number D. We use absolute values since predictability increases in both the following caes – when the fractal dimension becomes less than 1.5 and when it becomes greater than 1.5. In the former case, we have correlation (persistence) behaviour and in the latter case, anti-correlation (anti-persistence) behaviour. However, in either case, the process becomes more predictable. Thus, use of absolute values ensures that a process with $D = 1.3$ has the same predictability index as a process with $D = 1.7$.

The climate predictability index (PI_C) is defined as the collection of the above three indices:

$$\mathrm{PI}_C = (\mathrm{PI}_T, \mathrm{PI}_P, \mathrm{PI}_R). \tag{4}$$

If one of these indices is close to zero, then the corresponding process approximates the usual Brownian motion and is therefore unpredictable. If it is close to one, the process is very predictable. The rationale for introducing the climate predicatbility index is as follows. In this letter, we are interested in studying the inter-relationships between the three climatic components from the viewpoint of fractal dimensions. Hence, it is useful to have all three of them represented in a single index. Then it is easier to see how the three sub-indices change in relation to one another as the seasons change. Further, by introducing predictability indices instead of fractal dimensions, we focus on how predictable the process is. This is especially useful for precipitation.

One possibility that arises at this point is to somehow combine these three indices into a single number using an appropriate norm. But this may not be appropriate if even one of the processes is quite independent of the others. Therefore, for the present, we take the PI_C to be a collection of three indices. One important factor which has not been explicitly included in making up the PI_C is the geographical parameters of the location. However, in many cases it is possible that one of the above indices already includes the effect of geographical parameters implicitly. In such cases, the geographical parameters can be useful in explaining why one of the indices takes on the value it does. We will give examples of this later.

3. Applications

We now come to the actual calculation of PI_C and its possible uses. We concentrate on the Indian subcontinent because it lies at the heart of the classic monsoon region and is the area most sensitive to monsoon fluctuations. Data from twenty five measuring stations spread throughout India were studied. The temperature, pressure and precipitation time series for these stations were obtained from the Global Historical Climatology Network (GHCN) dataset [*Vose et. al.*, 1992]. In India, summer heat leads to low pressure over Tibetan plateau and northern India which induces a strong monsoon inflow (called the South-West monsoon) during the period June to September. In winter, the intense high pressure caused by the cooling of Siberia induces outflow of air that is partially blocked by Himalayan ranges [*Wasson*, 1995; *Lal et. al.*, 1994]. As a result, the Indian subcontinent is subjected to North-East monsoon during the months of October and November.

We first calculated separate mean temperature, mean pressure and mean precipitation time series from the GHCN data for the two periods given above. Fractal dimensions for these time series was then calculated using the R/S analysis described in the previous section. From these fractal dimensions, the PI_C was calculated (using the formula given in the previous section) for all stations — one for the June - September period and the other for October - November period.

We were particularly interested to see whether the precipitation predictability index changes with the seasons. This was indeed found to be the case in majority of the stations. However, to overcome possible errors involved in the calculation of PI_C from the time series, we restricted our attention to those stations where there was a significant change (greater than or equal to 0.4) in PI_R. Data regarding four stations (Madras, Veraval, Agra and Nagpur) which exhibited such a behaviour is summarized in Table 1. From the Table we see that for all four stations, precipitation which was fairly predictable during the June-September period has become totally unpredictable during the October-November period. Therefore, if fractal dimensions and predictability indices are different for different periods as in the above cases, it is most likely that the region is influenced by more than one climatic dynamics. Consequently, in such cases, calculations done using mean yearly data are suspect.

We now investigate the reasons for the change in PI_R for these four stations. The three main factors which influence precipitation are temperature, pressure and geographical parameters. In the case of Madras and Veraval (both coastal stations), there is no significant change in PI_P with the season. Hence it influences PI_R in a similar fashion in both periods. That is, as far as the influence of pressure is concerned, there should be no change in the predictability of precipitation with the season. However, predictability index for temperature, which can also influence PI_R, changes significantly. From Table 1, we see that predictability of temperature in the October - November period has decreased sharply for both stations when compared to the June - September period. This is what causes the decrease in predictability of precipitation. In the case of Agra and Nagpur (in interior India), the situation is different. Here, there is no change in PI_T with the season. However, there is a significant decrease in the predictability index for pressure which leads to a decrease in the corresponding index for precipitation. Thus, dynamic linkages that exist between the three climatic components are expressed in the above form when predictability of these components are studied.

Since South-West monsoon is most important for India, we next consider stations for which the precipitation during this monsoon is unpredictable (PI_R less than or equal to 0.1). There are three such stations (Bikaner, Ahmedabad and Pune), data for which is given in Table 2. For these stations, both temperature and pressure are on the whole predictable. However, two of the stations (Bikaner and Ahmedabad) are located in the north-western region of India – where the sinking limits of the South-West monsoon cell are also located [*Das*, 1962]. This leads to unpredictability in precipitation. The third station (Pune) is in the rain shadow region of the South-West monsoon [*Mishra and Rajaguru*, 1995] which again accounts for its low PI_R. Therefore, for these stations, it is their geographical parameter which strongly affects their PI_R.

Table 1. Change in PI_C with the season

Station	Location	PI_C for Jun-Sep	PI_C for Oct-Nov
Madras	13.0°N 80.2°E	$(0.6, 0.8, 0.4)$	$(0.1, 0.7, 0.0)$
Veraval	20.9°N 70.4°E	$(0.9, 0.4, 0.6)$	$(0.4, 0.4, 0.1)$
Agra	27.2°N 78.0°E	$(0.3, 0.7, 0.5)$	$(0.3, 0.2, 0.0)$
Nagpur	21.1°N 79.1°E	$(0.2, 0.6, 0.4)$	$(0.2, 0.2, 0.0)$

This Table lists data regarding stations in India that show significant change in precipitation predictability index from South-West monsoon (June - September) to North-East monsoon (November - December).

Table 2. Stations with low PI_R during SW monsoon

Station	Location	PI_C for Jun-Sep
Ahmedabad	23.1°N 72.6°E	$(0.8, 0.8, 0.0)$
Bikaner	28.0°N 73.3°E	$(0.6, 0.4, 0.1)$
Pune	18.5°N 73.9°E	$(0.4, 0.5, 0.0)$

This Table lists data regarding stations in India having low precipitation predictability index during South-West monsoon (June - September).

4. Discussions

In this letter, we have taken one step towards quantification of climatic uncertainities by proposing a climate predictability index. This index was obtained through a fractal dimensional analysis of the time series for three major components of a climate — temperature, pressure and precipitation. This index gives an indication of how predictable the climate is for a given station. It is particularly useful where a wet/dry seasonal pattern caused by monsoon (a planetary atmospheric circulation feature) dominates climate as in tropical and subtropical Asia, Australia and Africa. We demonstrated that predictabilty indices change significantly as the climatic dynamics change from one season to the other. Therefore, one has to be cautious in using a time series for yearly mean values (as is often done). Further, since the predictability index gives a single dimensionless number for each process, it can be used to roughly quantify the interplay between temperature, pressure and precipitation. We explicitly demonstrated how changes in predictability indices for either temperature or pressure affect the predictability of precipitation.

Moreover, the PI_C can be very useful when one is developing climatic models for a region. In climate prediction models, one looks for trends in the time series of climatic variables and correlations between them which can help specify the model. In these cases, one should avoid stations which have a low PI_C. Data from such a station would have random amplitude variations which are most probably caused by local conditions specific to that station. Such anomolous stations can skew the entire model.

We also believe that the shift in emphasis from fractal dimensions to predictability may by itself be useful since the latter concept is more intuitive. Instead of thinking in terms of fractal dimension and then making the association with its implications for the time series, it is much more straightforward to directly go to the concept of predictability. Finally, even though we have used the predictability indices in the context of climate, the basic concepts used would have applicabilty in other areas also.

Acknowledgments. One of us (DAS) would like to acknowledge support from CTS and the IISc-TIFR programme. We would also like to thank the referees for their useful comments.

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(Received September 23, 1996; revised December 13, 1996; accepted March 31, 1997.)