Discussion Meeting on Representation Theory 2019

Dates: December 14–16, 2019

Venue: LH 1, Department of Mathematics, Indian Institute of Science, Bangalore-560012

Titles and Abstracts

Amritanshu Prasad (IMSc, Chennai).

Title: Restrictions of Polynomial Representations of GL(n) to S(n) using character polynomials.

Abstract: We compute character polynomials of Weyl functors (which give rise to irreducible polynomial representations of GL(n) across n). We also give a simple algorithm to compute the inner product (as characters of S(n)) of character polynomials in general. Using formulas of Macdonald, and of Garsia and Goupil for character polynomials of Specht modules, we can then compute stable restriction coefficients which describe how an irreducible representations of GL(n) decomposes as a representation of S(n). We use our method to obtain generating functions for these coefficients and study some interesting special cases.

Anne Schilling (UC Davis, USA).

Title: The ubiquity of crystal bases

Abstract: Crystal bases originated from mathematical physics and the representation theory of quantum groups. I will give an introduction to the purely combinatorial treatment of crystal bases relying on Stembridge's characterization. The link between crystals and representation theory is made through Demazure crystals. We end with an application of crystal bases by providing a new crystal on decreasing factorizations of elements in the 0-Hecke monoid, which yields the Schur expansion of symmetric Grothendieck polynomials.

Apoorva Khare (IISc, Bangalore).

Title: Faces of highest weight modules and the universal Weyl polyhedron

Abstract: Let g be a complex semisimple Lie algebra, V a highest weight module, and write conv(V) for the convex hull of its set of weights. For V finite-dimensional, the faces and some of their inclusion relations were classified by Satake [Ann. of Math. 1960], Borel--Tits [Publ. IHES 1965], Vinberg [Izv. Nauk. 1990], and Casselman (1997) . The classification of the inclusions was completed for the adjoint representation by Cellini--Marietti [IMRN 2015].

In this talk,

(a) we will see the extension of this classification to all highest weight modules, over all (complex) Kac--Moody Lie algebras. This extends the Weyl polyope to `Weyl polyhedra' for arbitrary highest weight modules V.
(b) We present closed-form expressions for the dimension, Weyl group stabilizer, and vertex sets of faces, and the f-polynomial of conv(V).
(c) We assemble all Weyl polytopes/polyhedra (with a fixed integrability) into a universal family over a moduli space, and show it solves a moduli problem in convex geometry.
(d) We compute the localization of conv(V) along a face; this positively answers a question of Brion. (Partly joint with Tim Ridenour; partly with Gurbir Dhillon.

Arvind Ayyer (IISc, Bangalore).

Title: Bijective proofs of Schur polynomial factorisations

Abstract: In earlier work, Roger Behrend and I had shown that Schur polynomials of a wide class of partitions factor into symplectic and/or orthogonal group characters when half of the variables are specialised to be reciprocals of the other half. We now give combinatorial proofs of these factorisations using perfect matchings of weighted trapezoidal honeycomb graphs and their relation to Gelfand-Tsetlin patterns. An important ingredient in our proofs is Ciucu’s theorem for graphs with reflective symmetry. Time permitting, we will explain a generalisation of our result to skew Schur polynomials. A representation-theoretic proof of our results is suggested as a challenge to this audience. This is joint work with Ilse Fischer.

Pooja Singla (IISc, Bangalore).

Title: Steinberg Character for Symmetric Groups.

Abstract: Every finite group of Lie type has a special kind of irreducible character called Steinberg Character \chi which has the property that its dimension is equal to the cardinality of a Sylow-p subgroup of G and \chi(g) \neq 0 if and only if g is semisimple. During his lectures at ICTS, Dipendra Prasad asked if the converse of above is true, i.e. the existence of "Steinberg character" characterizes finite groups of Lie type. We shall discuss a solution to this problem for Symmetric groups. This is based on ongoing joint work with Digjoy Paul.

S. Viswanath (IMSc, Chennai).

Title: The Brylinski filtration and W-algebras

Abstract: Each finite dimensional irreducible representation V of a simple Lie algebra L admits a filtration induced by a principal nilpotent element of L. This, so-called Brylinski-Kostant filtration, can be restricted to the dominant weight spaces of V, and the resulting Hilbert series are very interesting q-analogs of weight multiplicity, first defined by Lusztig.

This picture can be extended to certain infinite dimensional Lie algebras L and representations V. We focus on special linear affine Lie algebras and their level 1 vacuum modules. In this case, we show how to produce a basis of the dominant weight spaces that is compatible with the Brylinski-Kostant filtration. This construction uses the W-algebra, a natural vertex algebra associated to L.

This is joint work with Sachin Sharma (IIT Kanpur) and Suresh Govindarajan (IIT Madras).

Tanusree Khandai (IISER, Mohali).

Title: Integrable Representations of Toroidal Lie algebras

Abstract: Toroidal Lie algebras are universal central extensions of multiloop Lie algebras. In the talk I will present a classification of integrable irreducible representations of toroidal Lie algebras with finite-dimensional weight spaces.

Vyjayanthi Chari (UC Riverside, USA, InfoSys Visiting Chair Professor-IISc).

Title: Introduction to cluster algebras and their connection with current algebras, Demazure modules and quantum affine algebras

Abstract: In these lectures I want to discuss the connections between cluster algebras, the representation theory of quantum affine algebras and Demazure modules in highest weight irreducible representations of positive level for affine Lie algebras.

The connection between cluster algebras and quantum affine algebras was developed by David Hernadez and Bernard Leclerc and is often referred to as the monoidal categorification of a cluster algebra. In the first lecture we will introduce the basic definitions in cluster theory along with some motivation for their study. In the second lecture we shall develop the connection with quantum affine algebras. We shall see how this connection helps us to identify important and not very well studied families of representations of quantum affine algebras. In the third lecture we shall relate these families to Demazure modules. The connection between quantum affine algebras and Demazure modules goes back to my work with Moura in 2005, followed by results of C-Loktev, Fourier-Littelmann, Naoi and others. But these connections are usually restricted to level one Demazure modules or to rectangular weights. The connection with cluster algebras shows the importance of non-rectangular weights. We conclude these lectures by showing how the study of these Demazure modules leads naturally to a new family of symmetric polynomials and their connection with Macdonald polynomials.

Short Talks (30 min).

Digjoy Paul (IMSc, Chennai).

Title: The multiset partition algebra

Abstract: We introduce a new diagram algebra $MP_\lambda(\xi)$ over $F[\xi]$ based on certain multiset partitions associated with a vector λ of nonnegative integers, where F is a field of characteristic 0. We provide a canonical embedding of $MP_\lambda(\xi)$ inside the partition algebra $P_{|\lambda|}(\xi)$ of **Jones and Martin**. As a consequence, we prove $MP_\lambda(\xi)$ is Cellular, Semisimple (except few cases).

Upon specializing ξ to a positive integer n, MP_λ(n) is in Schur-Weyl duality with the action of the symmetric group S_n on Sym^λ(F^n). We remark that when n ≥ 2|λ|, the dimension of an irreducible representation of MP_λ(n) is obtained by counting certain multiset partition tableaux. We give, extending a work of **Anne Schilling, Orellana, Zabrocki, et al,** RSK correspondence between multiset partition diagrams and pair of multiset partition tableaux. This supplies a bijective proof of a decomposition identity appears in this algebra. This talk is based on joint work with Shraddha Srivastava and Sridhar Narayanan.

G. V. K. Teja (IISc, Bangalore).

Title: Going between weights of integrable modules in root-steps

Abstract: In Lie theory the partial sum property says that for a root system in any Kac-Moody algebra, every positive root is an ordered sum of simple roots whose partial sums are all roots. We present a "parabolic" generalisation: if I is any nonempty subset of simple roots, then every root with positive I-height is an ordered sum of roots of I-height 1, whose partial sums are all roots. In fact we show this on the Lie algebra level, by showing that every root space is spanned by Lie words formed from root vectors of I-height 1. As an application, we provide a "minimal" description of wt(V), for every (non-integrable) simple highest weight module V over any Kac-Moody algebra g.

The partial sum property was extended in a different direction by Shrawan Kumar, to go between any two comparable weights in a finite-dimensional simple g-module. We extend this result to all simple highest weight modules, for g of finite or affine type.
