HW 11

1. Given a smooth manifold-without-boundary $M \subset \mathbb{R}^n$ and two points $p \neq q$, prove that there exist continuous functions $f, g : M \to \mathbb{R}$ such that
   (a) $f(p) = g(q) = 0$, $f(q) = g(p) = 1$.
   (b) For any coordinate parametrisation α, $f \circ \alpha$ and $g \circ \alpha$ are smooth.
2. Prove that any two parametrisations of a parametrised manifold are reparametrisations of each other.
3. Show that the closed upper hemisphere in n-dimensions is an $n-1$-dimensional smooth manifold-with-boundary whose boundary is the "boundary" $n-1$-dimensional sphere.
4. Prove that the set of orthogonal 3×3 matrices of determinant 1 (denoted as $SO(3)$) is a compact manifold-without-boundary in $\mathbb{R}^9 = Mat(3 \times 3, \mathbb{R})$. What is its dimension?
5. Suppose M is a compact manifold-with-boundary of dimension n in $\mathbb{R}^n$. Show that the topological and manifold interiors coincide.