HW 12

1. Let $v_1, \ldots, v_{n-1} \in \mathbb{R}^n$ be linearly independent vectors. Let V be the $n \times (n-1)$ matrix $V = [v_1\ v_2\ \ldots\ v_{n-1}]$. Let $c = \sum_i c_i e_i$ where $c_i = (-1)^{i-1} \det(v_1\ v_2 \ldots\ v_{i-1}\ v_i\ v_{i+1} \ldots)$ (where the i^{th} row is removed). Prove that c is the unique vector satisfying the following properties.

   (a) $c \neq 0$ and $c \perp v_i\ \forall\ i$.

   (b) $\det(c\ v_1\ v_2 \ldots) > 0$.

   (c) $\|c\|^2 = \det(V^T V)$.

2. (Optional) Complete the sketch of proof of Green's theorem.