HW 13

1. Complete the proof of the fact that a hypersurface is orientable iff it has a C^r unit normal vector field.

2. Suppose $M \subset \mathbb{R}^n$ is a C^r k-dimensional manifold-with-nonempty-boundary. Let $\vec{n} \in \mathbb{R}^n$ be a vector.

   (a) Suppose M is oriented. Let β be an oriented boundary C^r parametrisation near p. Define a C^r parametrisation γ of the boundary, i.e., $\gamma : U \subset \mathbb{R}^{k-1} \to \partial M$ to be orientation compatible with β if the ordered basis (for T_pM) $[-\frac{\partial \beta}{\partial x_k} \, \frac{\partial \gamma}{\partial u_1}(p) \, \ldots]$ is orientation-compatible with $[\frac{\partial \beta}{\partial x_1}(p) \, \frac{\partial \beta}{\partial x_2}(p) \, \ldots]$. Prove that the collection of all such γ that are orientation-compatible with the given orientation on M forms an orientation for ∂M.

   (b) Prove that this orientation is compatible with the standard orientation defined in the class.

3. Find an explicit orientation-compatible collection of parametrisations for the closed unit ball in $\mathbb{R}^4$ and an explicit collection of parametrisations for its boundary that are orientation-compatible with the standard orientation.