HW 14

1. Let V be a finite-dimensional real vector space of dimension n. Let $T : V \to V$ be a linear map. Prove that $T^* : \Lambda^n V \to \Lambda^n V$ is $T(\omega) = \det(T)\omega$.

2. Prove that the pullback of a smooth form field is a smooth form field.

3. Let $f : \mathbb{R}^3 \to \mathbb{R}^4$ be the smooth function $f(t, u, v) = (u^2, t^2, v, u)$ and let ω be the smooth 2-form field on $\mathbb{R}^4$ defined as ω at the point (x, y, z, w) equals $xdx \wedge dy + zdz \wedge dw + ydx \wedge dz$. Let η be the smooth one-form field on $\mathbb{R}^4$ defined as $xdy + ydz + zdw + wdx$. Prove explicitly (by calculating both sides) that $f^*(\omega \wedge \eta) = f^*\omega \wedge f^*\eta$.

4. Prove that $(f, g) \to f \wedge g$ is bilinear.

5. (Munkres Exercise 1 in chapter 28): Let $\vec{x}, \vec{y}, \vec{z} \in \mathbb{R}^5$. Let

$$F(\vec{x}, \vec{y}, \vec{z}) = 2x_2y_2z_1 + x_1y_5z_4$$

Write $Alt(F)$ in terms of the standard basis for differential 3-forms. Also write $Alt(F)(\vec{x}, \vec{y}, \vec{z})$ as a function.