HW 15

1. Let $\Omega \subset \mathbb{R}^2$ be an open set whose topological boundary is a collection of simple closed bounded parametrisated smooth curves that are smooth compact 1-manifolds. Then prove that $\bar{\Omega}$ is a smooth manifold-with-boundary (whose boundary is the topological boundary).

2. Let $\Omega \subset \mathbb{R}^3$ be an open set whose topological boundary is a collection of smooth compact surfaces. Then $\bar{\Omega}$ is a smooth compact 3-manifold-with-boundary whose boundary is the topological boundary).

3. Suppose η, ω are two smooth p-form fields on a smooth manifold-with-boundary M. Suppose $\alpha^*\omega = \alpha^*\eta$ for all smooth coordinate parametrisations α, then prove that $\eta = \omega$.

4. Let M be the orientable compact 3-manifold-with-boundary $\{(x, y, z, w) \in \mathbb{R}^4 \mid w \geq 0, x^2 + y^2 + z^2 + w^2 = 1\}$. Let $\omega = z^2 dx \wedge dy + x^2 dy \wedge dz + dx \wedge dw$ on $\mathbb{R}^4$. Choose an orientation on M (you need to specify the orientation you chose) and verify the generalised Stokes' theorem for ω $\left(\text{that is, } \int_M d\omega = \int_{\partial M} \omega\right)$ by *explicitly* calculating the left and right hand sides.

5. Prove that the wedge product of a closed form and an exact form is exact.