# HW 2

1. Let $F : \mathbb{R}^n \to \mathbb{R}$ be a homogeneous function of order $k$, i.e., $F(\lambda x) = \lambda^k F(x)$ for all $\lambda \in (0, \infty)$. Then prove Euler's identity: $x_1 F_{x_1} + x_2 F_{x_2} + \ldots + x_n F_{x_n} = kF$.

2. Let $F : Mat_{n\times n} \to Mat_{n\times n}$ be defined as $F(A) = A^3$. Prove that $F$ is differentiable and compute its derivative.

3. Let $a$ be an interior point of $U \subset \mathbb{R}^m$ and let $f : U \to \mathbb{R}$ be a function. If the partials $D_i f$ exist and are bounded in a neighbourhood of $a$, show that $f$ is continuous at $a$.

4. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ and $g : \mathbb{R}^3 \to \mathbb{R}^2$ be given by the equations $f(x) = (e^{2x_1+x_2}, 3x_2 - \cos(x_1), x_1^2 + x_2 + 2)$ and $g(y) = (3y_1 + 2y_2 + y_3^2, y_1^2 - y_3 + 1)$. If $F(x) = g \circ f(x)$, find $DF_{x=0}$ and if $G(y) = f \circ g(y)$, find $DG_{y=0}$.

5. Let $F(x, y) = f(x, y, g(x, y))$ where $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$ are differentiable. Find $DF$ in terms of the partials of $f$ and $g$. If $F(x, y) = 0\ \forall\ x, y$, find $g_x, g_y$ in terms of the partials of $f$.